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The Last Voting Rule Is Home: Complexity of Control by Partition of Candidates or Voters in Maximin Elections

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Abstract. One of the key topics of computational social choice is electoral control, which models certain ways of how an election chair can seek to influence the outcome of elections via structural changes such as adding, deleting, or partitioning either candidates or voters. Faliszewski and Rothe [13] have surveyed the rich literature on control, giving an overview of previous results on the complexity of the associated problems for the most important voting rules. Among those, only a few results were known for two quite prominent voting rules: Borda Count and maximin voting (a.k.a. the Simpson–Kramer rule). Neveling and Rothe [26, 25] recently settled the remaining open cases for Borda. In this paper, we solve all remaining open cases for the complexity of control in maximin elections all of which concern control by partition of either candidates or voters.

1 Introduction

Thirty years ago, in a series of seminal papers, Bartholdi et al. gave birth to the field of computational social choice by introducing and studying problems associated with winner determination for Dodgson and Kemeny elections [3], manipulation of elections [2, 1], and electoral control [4] in terms of their computational complexity. Later on, Hemaspaandra et al. [19, 22] pinpointed the complexity of determining Dodgson and Kemeny winners exactly, Faliszewski et al. [10, 12] introduced and studied bribery in elections, Conitzer et al. [7] studied coalitional weighted manipulation for the most important voting rules and also its destructive variant (where the manipulators' goal is not to make their favorite candidate win but to prevent their most despised candidate's victory), and Hemaspaandra et al. [20] studied destructive control. Among these research lines, we will here focus on *electoral control*.

Driven by the many applications of collective decision making (e.g., by voting) in artificial intelligence—ranging from automated scheduling [17] over recommender systems [15] to collaborative filtering [29], from computational linguistics [27] to information extraction [34], and from planning [9] to meta-searching the internet [8]—computational social choice has turned into an established area that is now a key topic of the major AI conferences (see, e.g., two recent papers in the AAAI Senior Member Track [23, 32]). This success story has been comprehensibly told in the *Handbook of Computational Social Choice* [6] and other books [31]. In particular, Faliszewski and Rothe [13] surveyed the state of the art in control (and bribery), summarizing the common control scenarios and the related complexity results for the most important voting rules.

However, results for two quite prominent voting rules in their chapter were scarce: By 2016, not much was known about the control complexity for the Borda Count and maximin voting, which is also known as the Simpson–Kramer rule. Borda is perhaps the most important rule within the class of scoring protocols: In Borda, voters rank the m candidates, the ith candidate in each ranking scores $m-i$ points, and whoever has the most points wins. Maximin, on the other hand, is a rule that, like Condorcet or Copeland, is based on pairwise comparisons: The candidates' maximin scores result from their worst pairwise comparison against other candidates and all candidates with the largest maximin score win.

While Neveling and Rothe [26, 25] recently settled the remaining open cases for Borda, in this paper we solve all remaining open cases regarding the control complexity in maximin elections. With our results, the "last voting rule is home" in the sense that we now have an almost[2] complete picture of the control complexity of all voting rules considered by Faliszewski and Rothe in their chapter [13]. All our results are NP-hardness results, that is, we will show that maximin is resistant to the corresponding types of control.

All these open issues for maximin voting concern control by partition of either candidates or voters. Previous results for maximin voting on the complexity of control by adding or deleting either candidates or voters are due to Faliszewski et al. [11], and some cases of destructive control by partition of candidates are due to Maushagen and Rothe [24]. We settle the remaining cases for maximin: constructive control by partition of candidates and constructive and destructive control by partition of voters.[3] In particular, control by partition of voters is very interesting, as it is a simple model of *gerrymandering* and therefore quite well motivated for application in the real world. It is also noteworthy that resistance to partition of candidates or voters typically is shown via the technically most involved proofs. Further, we have tried to simplify and unify our proofs as much as possible: Our eight NP-hardness results are shown via essentially only two constructions.

This paper is organized as follows. In Section 2, we give some background on elections and introduce some technical definitions that are helpful for our proofs. Our results for constructive control by partition of candidates in maximin elections are presented in Section 3 and our results for constructive and destructive control by partition of voters in Section 4. We conclude in Section 5.

2 Preliminaries

An election is given by a pair (C, V) with C being a set of candidates and V a profile of the voters' preferences over C. Each preference is a linear order over C. Identifying voters with the votes they cast,

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[2] For Schulze voting [33], it is still open how hard destructive control by adding and deleting candidates is.

[3] These control scenarios will be defined formally in Sections 3 and 4.

we will use the words *vote* and *voter* interchangeably. A vote of the form $b\ a\ c$ indicates that b is preferred to a and a to c.

Let $S \subseteq C$ be a subset of the candidates. When we write $\overrightarrow{S}$ in a vote, we mean a ranking of the candidates of S occurring in this vote in an arbitrary but fixed order; and when we write $\overleftarrow{S}$ in a vote, we mean their ranking in this vote in reverse order; and the order of the candidates from S does not matter in a vote when we simply write S in it. For example, letting $C = \{a, b, c, d\}$ and $S = \{b, c\}$ and assuming the lexicographic order of candidates, $d\ \overrightarrow{S}\ a$ means the vote $d\ b\ c\ a$; yet $d\ \overleftarrow{S}\ a$ means the vote $d\ c\ b\ a$; and $d\ S\ a$ could mean either of the votes $d\ b\ c\ a$ and $d\ c\ b\ a$.

To conveniently construct votes, for a set C of candidates and $c, d \in C$, let

$$W(c, d) = (c\ d\ \overrightarrow{C \setminus \{c, d\}},\ \overleftarrow{C \setminus \{c, d\}}\ c\ d)$$

be a pair of votes whose addition to a maximin election has the following effect on the scores of c and d: For such a pair, under maximin voting c gains two points in the head-to-head contest on d. For each other pair of candidates, they both gain one point in their head-to-head contest.

Some background from complexity theory is assumed, including standard notions such as the complexity classes P and NP, polynomial-time many-one-reducibility, and NP-hardness and NP-completeness. For more details, we refer to the textbooks by Garey and Johnson [14], Papadimitriou [28], and Rothe [30].

In our reductions for establishing NP-hardness, we will employ the following well-known NP-complete problems [14]. In ONE-IN-THREE-POSITIVE-3SAT, we are given a set X of boolean variables and a set $\mathcal{S}$ of clauses over X, each with exactly three unnegated literals, and we ask whether there is a truth assignment to the variables in X satisfying that in each clause of $\mathcal{S}$ exactly one literal is set to true. In EXACT-COVER-BY-3-SETS (X3C), we are given a set $B = \{b_1, \dots, b_{3k}\}$ and a family $\mathcal{S} = \{S_1, \dots, S_n\}$ of sets such that $S_i \subseteq B$ and $|S_i| = 3$ for all $S_i \in \mathcal{S}$, and the question is: Does there exist an exact cover of B, i.e., a subset $\mathcal{S}' \subseteq \mathcal{S}$ such that $|\mathcal{S}'| = k$ and $\bigcup_{S_i \in \mathcal{S}'} S_i = B$?

3 Constructive Control by Partition of Candidates

In this section, we consider the standard scenarios of constructive control by partition of candidates and solve all remaining open cases regarding the complexity of candidate control in maximin elections. In Table 1, we give an overview of all previously known and new results on the complexity of candidate control for maximin. We start by defining in Section 3.1 the problems we consider and then prove our results in Section 3.2.

3.1 Problem Definitions and Overview of Results

We first consider constructive control by partition of candidates, as defined by Bartholdi et al. [4] for any given voting rule $\mathcal{E}$ (which here will always be maximin). In this scenario, the election (C, V) is held in two rounds and we assume that the chair has the power to subdivide the candidates into two groups, C_1 and C_2, and the winners of the first-round subelection (C_1, V) (where we tacitly assume that the votes in V are restricted to the candidates in C_1) run against the candidates in C_2, i.e., all members of C_2 get a bye to the final round.

We will adopt the so-called *unique-winner model* (as Bartholdi et al. [4] did), which means that for a control action to be successful, it is required that the distinguished candidate is the *only* winner. By contrast, in the *nonunique-winner model*, which also has been studied intensively for control problems [13, 5], it would be enough that the distinguished candidate is one among possibly several winners for a control action to be successful. Note that our results, even though expressed in the unique-winner model, hold also in the nonunique-winner model, as can be shown by slight modifications of our proofs.[4]

We will use the following tie-handling rules due to Hemaspaandra et al. [21]: According to *ties-promote* (TP) all winners of a first-round subelection will move forward to the final round and according to *ties-eliminate* (TE) only a unique first-round subelection winner moves forward to the final round (i.e., if there are two or more first-round subelection winners, they eliminate each other and no one moves to the final round from this first-round subelection).[5]

Now we are ready to define the first decision problem, $\mathcal{E}$-CONSTRUCTIVE-CONTROL-BY-PARTITION-OF-CANDIDATES-TP for voting rule $\mathcal{E}$, which given an election (C, V) and a distinguished candidate $p \in C$, asks whether we can partition C into C_1 and C_2 such that p is the unique $\mathcal{E}$ winner of the two-round election where the winners of the first-round subelection (C_1, V) run against all members of C_2 in a final round (with the votes from V in all subelections appropriately restricted to the participating candidates.

This problem name is abbreviated by $\mathcal{E}$-CCPC-TP. The related problem $\mathcal{E}$-CONSTRUCTIVE-CONTROL-BY-RUNOFF-PARTITION-OF-CANDIDATES-TP ($\mathcal{E}$-CCRPC-TP, for short; also due to Bartholdi et al. [4]) is defined similarly, except that now we have two first-round subelections, (C_1, V) and (C_2, V), and the winners of both proceed to the final runoff. With the other tie-handling rule, ties-eliminate, we receive the corresponding problems $\mathcal{E}$-CCPC-TE and $\mathcal{E}$-CCRPC-TE.

The destructive variants of these four problems, due to Hemaspaandra et al. [21], are defined analogously, except that the chair's goal now is to prevent the victory of the distinguished candidate. We abbreviate the corresponding problems by $\mathcal{E}$-DCPC-TP, $\mathcal{E}$-DCRPC-TP, $\mathcal{E}$-DCPC-TE, and $\mathcal{E}$-DCRPC-TE.[6]

Further, Bartholdi et al. [4] and Hemaspaandra et al. [21] introduced and studied for various voting rules the notions of constructive and destructive control by adding candidates (CCAC and DCAC), by adding an unlimited number of candidates (CCAUC and DCAUC), and by deleting candidates (CCDC and DCDC). As we do not study these control scenarios here, we refrain from defining them formally, instead referring to the work of Bartholdi et al. [4] and Hemaspaandra et al. [21] and in particular to the work of Faliszewski et al. [11] who obtained results for them in maximin elections. Table 1 gives an overview of all previous complexity results for candidate control in maximin elections (which are due to Faliszewski et al. [11] and Maushagen and Rothe [24]) as well as the new complexity results for candidate control in maximin elections established in this paper.

A voting rule $\mathcal{E}$ maps each election (C, V) to a subset of can-

[4] In fact, the constructions need not be changed, as in the yes-instances p will always win alone, whereas in the no-instances p will never even win. That is, the stronger condition of the unique-winner or the nonunique-winner model will always be satisfied. All that needs to be changed in the proofs for them to work also in the nonunique-winner model are minor modifications of the wording in the argumentation.

[5] Note that the unique-winner model better fits the TE rule and the nonunique-winner model better fits the TP rule.

[6] Hemaspaandra et al. [18] noted that, depending on whether we use TP or TE and on what winner model we choose (i.e., either the unique-winner or the nonunique-winner model), DCPC and DCRPC can be identical problems. Specifically, in the unique-winner model, we have DCRPC-TE = DCPC-TE, and in the nonunique-winner model, we have DCRPC-TE = DCPC-TE and DCRPC-TP = DCPC-TP.

CAUC C	CAUC D	CAC C	CAC D	CDC C	CDC D	CPC-TE C	CPC-TE D	CPC-TP C	CPC-TP D	CRPC-TE C	CRPC-TE D	CRPC-TP C	CRPC-TP D
V♡	V♡	R♡	V♡	V♡	V♡	**R**	V♠	**R**	V♠	**R**	V♠	**R**	V♠

Table 1. Overview of complexity results for candidate control in maximin elections. R means resistance and V means vulnerability. Results in boldface are established in this paper, and previous results are due to Faliszewski et al. [11] (marked by ♡) and due to Maushagen and Rothe [24] (marked by ♠).

didates, the *winners* of the election. We focus on the *maximin* voting rule (a.k.a. *Simpson–Kramer's rule*), which is based on pairwise comparisons. Given an election (C, V), for any two candidates $c, d \in C$, we denote the number of voters preferring c to d by $N_V(c, d)$. The *maximin score of candidate* c then is

$$\text{score}_{(C,V)}(c) = \min_{d \in C \setminus \{c\}} N_V(c, d) - N_V(d, c),$$

and whoever has the largest maximin score wins the election. In the following, we will omit the subscripts and simply write $\text{score}(c)$ and $N(c, d)$ if the meaning is clear from the context. A *Condorcet winner* is a candidate who wins each pairwise comparison; thus a Condorcet winner is always a maximin winner.

A voting rule $\mathcal{E}$ is said to be *immune to a control type* $\mathfrak{C}$ (such as constructive control by partition of candidates when ties promote) if it is never possible for the chair to reach her control goal; otherwise, it is said to be *susceptible to* $\mathfrak{C}$. Note that maximin is easily seen to be susceptible to every control type considered in this paper. If $\mathcal{E}$ is susceptible to $\mathfrak{C}$, we are interested in the computational complexity of the associated control problem (such as $\mathcal{E}$-CCPC-TP). We say that $\mathcal{E}$ is *vulnerable to* $\mathfrak{C}$ if $\mathcal{E}$ is susceptible to $\mathfrak{C}$ and the control problem corresponding to $\mathfrak{C}$ can be solved in polynomial time, and we say $\mathcal{E}$ is *resistant to* $\mathfrak{C}$ if $\mathcal{E}$ is susceptible to $\mathfrak{C}$ and the corresponding control problem is NP-hard.

3.2 Results and Proofs

While maximin is vulnerable to destructive control by partition and runoff partition of candidates in model TP and TE [24], we will now show that it is resistant to *constructive* control by partition and runoff partition of candidates with both tie-handling rules, TP and TE. For each of these four problems, we can use the same reduction.

Theorem 3.1. *For maximin elections, each of the problems* CCPC-TP*,* CCRPC-TP*,* CCPC-TE*, and* CCRPC-TE *is* NP*-complete.*

Proof. Membership of all four problems in NP is obvious. To show NP-hardness, we reduce from ONE-IN-THREE-POSITIVE-3SAT. Let $(X, \mathcal{S})$ be a ONE-IN-THREE-POSITIVE-3SAT instance with $X = \{x_1, \ldots, x_m\}$ and $\mathcal{S} = \{S_1, \ldots, S_n\}$, where $S_i = \{x_{i,1}, x_{i,2}, x_{i,3}\} \subseteq X$ and $|S_i| = 3$ for each $1 \leq j \leq n$. In the following, we use each $x \in X$ and each $S \in \mathcal{S}$ both as part of the given instance of ONE-IN-THREE-POSITIVE-3SAT and as the candidates of the election that is part of the constructed instance of any of the four control-by-partition problems. It will always be clear from the context what meaning is intended.

Specifically, from $(X, \mathcal{S})$ we construct an election (C, V) with the set $C = \{p, d, w\} \cup X \cup \mathcal{S} \cup R \cup T$ of candidates with $R = \{r_{i,j} \mid 1 \leq i \leq n, 1 \leq j \leq 3\}$ and $T = \{t_{i,j} \mid r_{i,j} \in R\}$.

The list V of votes is constructed as follows, where we write $[n]$ for the set $\{1, \ldots, n\}$ for $n \in \mathbb{N}$:

#	preference	for each
1	$W(r, p)$	$r \in R$
1	$W(t, p)$	$t \in T$
1	$W(w, S)$	$S \in \mathcal{S}$
1	$W(w, r)$	$r \in R$
2	$W(p, x)$	$x \in X$
2	$W(p, d)$	
4	$W(r, x)$	$r \in R, x \in X$
4	$W(x_{i,j}, t_{i,j})$	$i \in [n], j \in [3]$
4	$W(t_{i,j}, r_{i,j})$	$i \in [n], j \in [3]$
4	$W(x_{i,j}, t_{i,(j+1) \bmod 3})$	$i \in [n], j \in [3]$
$1 + j - i$	$W(S_i, S_j)$	$1 \leq i < j \leq n$
$n + 1$	$W(S, r)$	$S \in \mathcal{S}, r \in R$
$n + 1$	$W(S, d)$	
$n + 2$	$W(d, w)$	
$n + 3$	$W(w, x)$	$x \in X$
$n + 3$	$W(x, S)$	$S \in \mathcal{S}, x \in S$
$n + 3$	$W(w, t)$	$t \in T$
$n + 3$	$W(t, d)$	$t \in T$
$n + 3$	$W(r, d)$	$r \in R$
$n + 4$	$W(w, p)$	
$n + 4$	$W(S, p)$	$S \in \mathcal{S}$

This construction is sketched in Figures 1 and 2. In particular, Figure 2 shows the subgraph among the candidates from the sets X, R, T, and $S_i \in \mathcal{S}$ for fixed i, where a directed edge between two candidates, say pointing from a to b, means that a wins the pairwise comparison against b. The edges are weighted and their positive integer weights give the numbers of how often preference $W(a, b)$ occurs in the construction. Doubling these weights gives the surplus indicating how strongly a wins against b. If an edge starts from a gray rectangle, this means that *all* candidates from this set beat the candidates the edge points to, and similarly so the other way around: an edge pointing to a gray rectangle means that *all* candidates in this set are beaten by the candidate this edge originates from. In the graph shown in Figure 1, on the other hand, these subgraphs from Figure 2 are only roughly adumbrated (each framed by a dotted line).

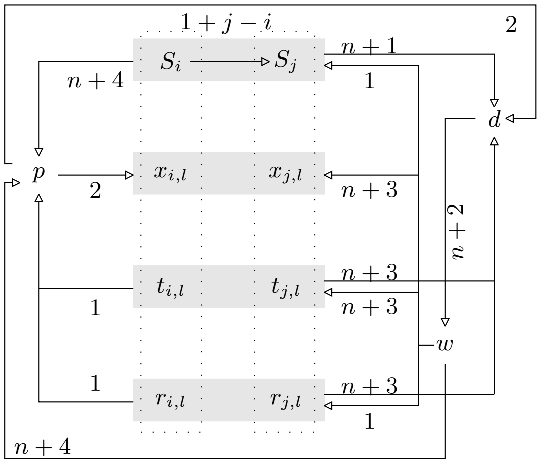

Figure 1. Construction in the proof of Theorem 3.1: Overview

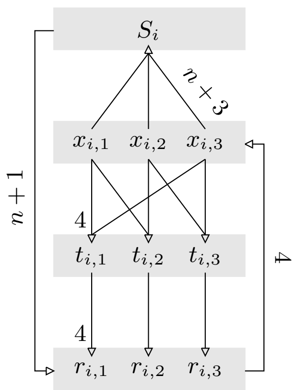

Figure 2. Construction in the proof of Theorem 3.1: subgraph for one S_i

Let $\Pi \in \{\text{CCRPC-TP}, \text{CCPC-TP}, \text{CCRPC-TE}, \text{CCPC-TE}\}$. We show: $(X, \mathcal{S})$ is a yes-instance of ONE-IN-THREE-POSITIVE-3SAT if and only if (C, V, p) is a yes-instance of maximin-Π.

From left to right, let $(X, \mathcal{S})$ be a yes-instance of ONE-IN-THREE-POSITIVE-3SAT. Then there is a subset $U \subseteq X$ such that for each clause S_i we have $|U \cap S_i| = 1$. Partition the set C of candidates into C_1 and C_2 with

$$\begin{aligned} C_1 &= \{d, w\} \cup \mathcal{S} \cup U \quad \text{and} \\ C_2 &= \{p\} \cup R \cup T \cup X \setminus U. \end{aligned}$$

Let us start with the case where we have two first-round subelections (i.e., with constructive control by runoff partition of candidates). In the subelection (C_1, V), each candidate $x \in U$ and each candidate $S \in \mathcal{S}$ has a score of $-2(n+3)$, w has a score of $-2(n+2)$, and d wins the subelection with a score of $-2(n+1)$. Since d is the unique winner of (C_1, V), d will move forward to the final round, regardless of the tie-breaking rule. In the subelection (C_2, V), the candidate p gets a score of -2, while each other candidate gets a score of -8. Since p is the unique winner of the subelection (C_2, V), p will move forward to the final round, regardless of the tie-breaking rule. In the final round, p faces only d and only p wins this head-on-head contest and thus the election.

Let us now consider the case of constructive control by partition of candidates where we have only one first-round subelection, (C_1, V), and all candidates from C_2 move directly forward to the final round. Compared with the subelection (C_2, V), the occurrence of d does not affect the score of any candidate. Since d has a score smaller than -4, p is again the only winner of the final election.

It follows that (C, V, p) is a yes-instance of maximin-Π for each $\Pi \in \{\text{CCPC-TP}, \text{CCRPC-TP}, \text{CCPC-TE}, \text{CCRPC-TE}\}$, completing the proof of the desired equivalence from left to right.

Conversely, from right to left, assuming that $(X, \mathcal{S})$ is a no-instance of ONE-IN-THREE-POSITIVE-3SAT, we will show that the constructed instance (C, V, p) is a no-instance of maximin-Π for each $\Pi \in \{\text{CCPC-TP}, \text{CCRPC-TP}, \text{CCPC-TE}, \text{CCRPC-TE}\}$ as well, i.e., we show that p cannot be made a unique winner of the two-stage election resulting from any possible partition of the candidate set (with or without runoff and for both tie-handling rules).

The worst pairwise comparison for p is between p and w as well as between p and each candidate $S \in \mathcal{S}$. Thus, if p were to face any of these candidates in a first-round subelection or in the final round, p would not win according to this partition of C. Therefore, we have to show that for each partition of candidates, where w and all $S \in \mathcal{S}$ participate in (C_1, V) and p is in (C_2, V), p does not win the election.

We consider all remaining partitions below.

Case 1: Let $C_1 = \{w\} \cup \mathcal{S} \cup T' \cup R' \cup U$ with $T' \subseteq T$, $R' \subseteq R$, and $U \subseteq X$. Candidate w wins each head-to-head contest, so w is the unique winner and can move forward to the final election. It follows that p can not win the election.

Case 2: Let $C_1 = \{d, w\} \cup \mathcal{S} \cup U$ with $U \subseteq X$. We consider two subcases.

Case 2.1: For each $S \in \mathcal{S}$, it is $S \cap U \neq \emptyset$. Since, we started with a no-instance of ONE-IN-THREE-POSITIVE-3SAT, there is at least one S_i such that $x_{i,j}, x_{i,j+1 \bmod 3} \in C_1$ with $1 \leq j \leq 3$. Each $S \in \mathcal{S}$ and each $x \in U$ has a score of $-2(n+3)$, w has a score of $-2(n+2)$ and d has a score of $-2(n+1)$. Therefore, d is the unique winner of the subelection and can move forward to the final round. Now, we have to destinguish between CCRPC and CCPC. We start with CCRPC. In (C_2, V), we have $C_2 = \{p\} \cup T \cup R \cup X \setminus U$. Candidate $t_{i,j+1 \bmod 3}$ has a score of 0, whereas p has a score of -2. It follows that p can not move forward to the final election. Let us now consider where we have only one subelection, CCPC. Since d is the unique winner of the first subelection, the final runoff is $(C_2 \cup \{d\}, V)$. Compared with the subelection (C_2, V), the occurrence of d does not affect the score of any candidate. It follows that p can not win the final election.

Case 2.2: It exists a $S \in \mathcal{S}$ with $S \cap U = \emptyset$. Let $\mathcal{S}' = \{S \mid S \cap U = \emptyset\} \subset \mathcal{S}$. Each $S_i \in \mathcal{S}'$ has a score of $-2i$, each $S_i \in \mathcal{S} \setminus \mathcal{S}'$ and each $x \in U$ has a score of $-2(n+3)$, d has a score of $-2(n+1)$, and w has a score of $-2(n+2)$. The candidate $S_i \in \mathcal{S}'$ with the lowest subscript wins the election.

Case 3: Let $C_1 = \{d, w\} \cup \mathcal{S} \cup T' \cup R'$ with $T' \subseteq T$, $R' \subseteq R$. Each S_i has a score of $-2i$ and each other candidate has a score lower than or equal to $-2(n+1)$. Therefore, S_1 is the unique winner of the subelection and can move forward to the final runoff, such that p can not win the election.

Case 4: Let $C_1 = \{d, w\} \cup \mathcal{S} \cup T' \cup R' \cup U$ with $T' \subseteq T$, $R' \subseteq R$. For $U = \emptyset$, we have Case 3 and for $T' \cup R' = \emptyset$, we have Case 2. We consider two subcases.

Case 4.1: It is $S \cap U \neq \emptyset$ for each $S \in \mathcal{S}$. Candidate w is the unique winner with a score of $-2(n+2)$ while each other candidate has a score of $-2(n+3)$. Thus, p can not win the election.

Case 4.2: It exists $S \in \mathcal{S}$ such that $S \cap U = \emptyset$. Let $\mathcal{S}' = \{S \mid S \cap U = \emptyset\} \subset \mathcal{S}$. Each $S_i \in \mathcal{S}'$ has a score of $-2i$, w has a score of $-2(n+2)$ and each other candidate has a score of $-2(n+3)$. The candidate $S_i \in \mathcal{S}'$ with the lowest subscript is the unique winner of the subelection. It follows that p can not win the election.

It follows that (C, V, p) is a no-instance of maximin-Π for each $\Pi \in \{\text{CCRPC-TP}, \text{CCPC-TP}, \text{CCRPC-TE}, \text{CCPC-TE}\}$. ❑

4 Control by Partition of Voters

We now turn to constructive and destructive control by partition of voters, again with the tie-handling rules TP and TE, thus solving all remaining open cases for the complexity of voter control in maximin elections. In Table 2, we give an overview of all previously known and new results on the complexity of voter control for maximin.

We start by defining in Section 4.1 the problems we consider and will then prove our results in Section 4.2.

4.1 Problem Definitions and Overview of Results

These problems have also been introduced and studied by Bartholdi et al. [4] and Hemaspaandra et al. [21]. In control by partition of voters, the election (C, V) is again held in two rounds but now we assume that the chair has the power to subdivide the *voters* into two groups, V_1 and V_2, and the winners of the two first-round subelections (C, V_1) and (C, V_2) run against each other in a final round. This can be seen as a very simple model of *gerrymandering*.

Specifically, for any given voting rule $\mathcal{E}$ (which again will always be maximin), we give the formal definition of one of the decision problems we study in detail: In $\mathcal{E}$-CONSTRUCTIVE-CONTROL-BY-PARTITION-OF-VOTERS-TP, we are given an election (C, V) and a distinguished candidate $p \in C$, and we ask whether V can be partitioned into V_1 and V_2 such that p is the unique $\mathcal{E}$ winner of the two-round election where the winners of the two first-round subelections, (C, V_1) and (C, V_2), run against each other in a final round.

We abbreviate this problem by $\mathcal{E}$-CCPV-TP. The related problem for the other tie-handling rule, ties-eliminate, is $\mathcal{E}$-CCPV-TE, and the two destructive variants are $\mathcal{E}$-DCPV-TP and $\mathcal{E}$-DCPV-TE.

Bartholdi et al. [4] and Hemaspaandra et al. [21] also introduced and studied for various voting rules the notions of constructive and destructive control by adding voters (CCAV and DCAV) and by deleting voters (CCDV and DCDV). Again, we refrain from defining these formally, as we won't study these control scenarios here, and we instead refer to the work of Bartholdi et al. [4] and Hemaspaandra et al. [21] and in particular to that of Faliszewski et al. [11] who obtained results for them in maximin elections. Table 2 gives an overview of all previous complexity results for voters control in maximin elections (due to Faliszewski et al. [11]).

For notational convenience and to simplify our proofs, we use a slightly different scoring function in this section. Given an election (C, V) and a candidate $c \in C$, define $\text{Score}_{(C,V)}(c) = \min_{d \in C \setminus \{c\}} N_V(c, d)$, again omitting the subscript if (C, V) is clear from the context. Note that

$$\begin{aligned} N_V(c,d) - N_V(d,c) &= N_V(c,d) - (|V| - N_V(c,d)) \\ &= 2N_V(c,d) - |V|, \end{aligned}$$

so this is simply a linear shift compared with $\text{score}_{(C,V)}(c)$.

CAV C	CAV D	CDV C	CDV D	CPV-TE C	CPV-TE D	CPV-TP C	CPV-TP D
R$^{\heartsuit}$	R$^{\heartsuit}$	R$^{\heartsuit}$	R$^{\heartsuit}$	**R**	**R**	**R**	**R**

Table 2. Overview of complexity results for voter control in maximin elections. R means resistance and V means vulnerability. Results in boldface are established in this paper, and previous results are due to Faliszewski et al. [11] (marked by ♡).

4.2 Results and Proofs

We first consider control by partition of voters when ties promote.

Theorem 4.1. *maximin*-CCPV-TP *and maximin*-DCPV-TP *are* NP-*complete.*

Proof. Membership of these two problems in NP again is obvious. To show NP-hardness, we reduce from a variant of X3C. Let $(B, \mathcal{S})$ be an X3C instance, where $B = \{b_1, \ldots, b_{3k}\}$, $\mathcal{S} = \{S_1, \ldots, S_n\}$, and $S_i = \{b_{i,1}, b_{i,2}, b_{i,3}\}$ for each $S_i \in \mathcal{S}$. Without loss of generality, we may assume that $k > 6$. Furthermore, we may assume that each $b_j \in B$ is contained in exactly three sets $S_i \in \mathcal{S}$; thus $|B| = |\mathcal{S}| = n$. That X3C even with this restriction is still NP-hard was shown by Gonzalez [16].

From $(B, \mathcal{S})$, we construct an election with the set $C = \{p, d, w\} \cup B$ of candidates and with the distinguished candidate p for the constructive case and the distinguished candidate w for the destructive case. The list V of votes is constructed as follows:

#	preference	for each
$2k-1$	$w\ p\ B\ d$	
$4k-2$	$w\ d\ B\ p$	
1	$w\ B\ p\ d$	
1	$B \setminus S_i\ p\ d\ w\ S_i$	$S_i \in \mathcal{S}$
k	$B\ p\ w\ d$	
$2k$	$B\ d\ w\ p$	

In total, we have $12k - 2$ votes. In these votes, the candidates from B are shifted cyclically such that, in particular:

- the first vote is $w\ p\ b_1\ b_2\ \cdots\ b_{3k-1}\ b_{3k}\ d$,
- the second vote is $w\ p\ b_{3k}\ b_1\ b_2\ \cdots\ b_{3k-1}\ d$, and
- the last vote is $b_4\ \cdots\ b_{3k}\ b_1\ b_2\ b_3\ d\ w\ p$.

However, the description of the construction is not finished yet; there still is a final twist to take notice of, regarding the fourth row of the construction, which gives one vote of the form $B \setminus S_i\ p\ d\ w\ S_i$ for each $S_i \in \mathcal{S}$. When constructing these $3k$ votes, let us first pretend that in this fourth row there were a vote of the form $B\ p\ d\ w$ (with the elements of B cyclically shifted as described above). But then we change each such vote $B\ p\ d\ w$ in the following way. For the ith vote $B\ p\ d\ w$, we shift $b_{i,1}$, $b_{i,2}$, and $b_{i,3}$ (i.e., the elements of S_i) from their current position to the end of the vote, while keeping the relative order among $b_{i,1}$, $b_{i,2}$, and $b_{i,3}$. For illustration, consider the following example. Suppose $S_1 = \{b_1, b_2, b_3\}$. After pretending that we have only votes of the form $B\ p\ d\ w$ instead of $B \setminus S_i\ p\ d\ w\ S_i$, we have a vote $b_3\ b_4\ \cdots\ b_{3k}\ b_1\ b_2\ p\ d\ w$ according to the cyclic shifts. This vote is then changed into: $b_4\ \cdots\ b_{3k}\ p\ d\ w\ b_3\ b_1\ b_2$.

Example 4.2. *As an illustration of the entire construction, consider the following example. Note that, for the sake of convenience and readability, we assume $k = 2$ in this example (even though we actually require $k > 6$ for the proof to work). Let $B = \{b_1, \ldots, b_6\}$ and $\mathcal{S} = \{S_1, \ldots, S_6\}$ with $S_1 = \{b_1, b_2, b_3\}$, $S_2 = \{b_4, b_5, b_6\}$, $S_3 = \{b_2, b_3, b_6\}$, $S_4 = \{b_2, b_4, b_5\}$, $S_5 = \{b_1, b_3, b_4\}$, and $S_6 = \{b_1, b_5, b_6\}$. According to our construction, we first get the following votes.*

From the first row of our vote list, we obtain three votes:

$$w\ p\ b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ d,$$
$$w\ p\ b_6\ b_1\ b_2\ b_3\ b_4\ b_5\ d,$$
$$w\ p\ b_5\ b_6\ b_1\ b_2\ b_3\ b_4\ d.$$

From the second row of our vote list, we obtain six votes:

$$w\ d\ b_4\ b_5\ b_6\ b_1\ b_2\ b_3\ p,$$
$$w\ d\ b_3\ b_4\ b_5\ b_6\ b_1\ b_2\ p,$$
$$w\ d\ b_2\ b_3\ b_4\ b_5\ b_6\ b_1\ p,$$
$$w\ d\ b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ p,$$
$$w\ d\ b_6\ b_1\ b_2\ b_3\ b_4\ b_5\ p,$$
$$w\ d\ b_5\ b_6\ b_1\ b_2\ b_3\ b_4\ p.$$

From the third row of our vote list, we obtain one vote:

$$w\ b_4\ b_5\ b_6\ b_1\ b_2\ b_3\ p\ d.$$

Note that the cyclic shifts of the elements of B spread across the single lines of our vote list. Now, from the fourth row of our vote list, we obtain six further votes, and now the cyclic shifts of the elements of B are slightly tampered with as we have explained above. Pretending we had the vote $B\ p\ d\ w$ in the fourth row of our vote list, then the first vote would be $b_3\ b_4\ b_5\ b_6\ b_1\ b_2\ p\ d\ w$. According to $B \setminus S_1\ p\ d\ w\ S_1$ with $S_1 = \{b_1, b_2, b_3\}$, this gives the vote: $b_4\ b_5\ b_6\ p\ d\ w\ b_3\ b_1\ b_2$. That is, the six votes corresponding to the fourth row are changed from

$$b_3\ b_4\ b_5\ b_6\ b_1\ b_2\ p\ d\ w,$$
$$b_2\ b_3\ b_4\ b_5\ b_6\ b_1\ p\ d\ w,$$
$$b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ p\ d\ w,$$
$$b_6\ b_1\ b_2\ b_3\ b_4\ b_5\ p\ d\ w,$$
$$b_5\ b_6\ b_1\ b_2\ b_3\ b_4\ p\ d\ w,$$
$$b_4\ b_5\ b_6\ b_1\ b_2\ b_3\ p\ d\ w$$

to the six votes where the three candidates corresponding to the $S_i \in \mathcal{S}$ are moved to the end of these six votes, respecting their relative order:

$$b_4\ b_5\ b_6\ p\ d\ w\ b_3\ b_1\ b_2,$$
$$b_2\ b_3\ b_1\ p\ d\ w\ b_4\ b_5\ b_6,$$
$$b_1\ b_4\ b_5\ p\ d\ w\ b_2\ b_3\ b_6,$$
$$b_6\ b_1\ b_3\ p\ d\ w\ b_2\ b_4\ b_5,$$
$$b_5\ b_6\ b_2\ p\ d\ w\ b_1\ b_3\ b_4,$$
$$b_4\ b_2\ b_3\ p\ d\ w\ b_5\ b_6\ b_1.$$

Next, from the fifth row of our vote list, we obtain two votes:

$$b_3\ b_4\ b_5\ b_6\ b_1\ b_2\ p\ w\ d,$$
$$b_2\ b_3\ b_4\ b_5\ b_6\ b_6\ p\ w\ d.$$

Finally, from the sixth row of our vote list, we obtain four votes:

$$b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ d\ w\ p,$$
$$b_6\ b_1\ b_2\ b_3\ b_4\ b_5\ d\ w\ p.$$
$$b_5\ b_6\ b_1\ b_2\ b_3\ b_4\ d\ w\ p,$$
$$b_4\ b_5\ b_6\ b_1\ b_2\ b_3\ d\ w\ p.$$

The reduction can obviously be computed in polynomial time. Due to the cyclic shifts, we have $N(b_1, b_{3k}) \leq 7$ and $N(b_i, b_{i-1}) \leq 7$ for each i, $2 \leq i \leq 3k$. Therefore, $\mathrm{Score}(b) \leq 7$ for each $b \in B$. For the remaining candidates, we have the following pairwise comparisons:

$N(\downarrow, \rightarrow)$	p	w	d	$b \in B$
p	–	$4k$	$6k$	$2k+2$
w	$8k-2$	–	$7k-2$	$6k+1$
d	$6k-2$	$5k$	–	$4k+1$

We notice that w is the unique maximin winner and also a Condorcet winner of the election.

We claim that $(B, \mathcal{S})$ is in X3C if and only if (C, V, p) is a yes-instance of maximin-CCPV-TP (respectively, (C, V, w) is a yes-instance of maximin-DCPV-TP).

From left to right, let $(B, \mathcal{S})$ be a yes-instance of X3C. Then there is a subset $\mathcal{S}' \subseteq \mathcal{S}$ with $|\mathcal{S}'| = k$ and $\bigcup_{S_i \in \mathcal{S}'} S_i = B$. Partition V into V_1 and V_2, where V_1 contains all votes of the form $w\ p\ B\ d$, as well as $B\ p\ w\ d$ and the k votes $B \setminus S_i\ p\ d\ w\ S_i$ and $S_i \in \mathcal{S}'$. The remaining votes are in V_2. In (C, V_1), we have:

$N(\downarrow, \rightarrow)$	p	w	d	$b \in B$
p	–	$2k$	$4k-1$	$2k$
w	$2k-1$	–	$3k-1$	$2k$
d	0	k	–	1

So p is the sole winner of the first first-round subelection and moves forward to the final round. In (C, V_2), we have:

$N(\downarrow, \rightarrow)$	p	w	d	$b \in B$
p	–	$2k$	$2k+1$	2
w	$6k-1$	–	$4k-1$	$4k+1$
d	$6k-2$	$4k$	–	$4k$

The candidate d has with $4k$ the highest score and is the only winner of the second first-round subelection.

The final runoff thus is $(\{p, d\}, V)$. We have $6k$ of $12k - 2$ votes who prefer p to d such that p is the unique winner of the runoff. It follows that (C, V, p) (respectively, (C, V, w)) is a yes-instance of maximin-CCPV-TP (respectively, maximin-DCPV-TP), as desired.

From right to left, assuming that $(B, \mathcal{S})$ is a no-instance of X3C, we will show that for each partition of the voters, w is always the unique winner of the election.

A Condorcet winner in an election (C, V) remains a Condorcet winner for each subset $C' \subseteq C$.[7] Since w is a Condorcet winner in (C, V), it is necessary that w cannot move forward to the final round, or else w's overall victory cannot be prevented.

We have $6k - 2$ out of $12k - 2$ votes who prefer w the most. In the following, we denote the number of voters with a vote of the form $w\ \cdots$ in the subelection (C, V_i) with ℓ_i for $i \in \{1, 2\}$ when V is partitioned into V_1 and V_2. Note that $N_{V_i}(w, c) \geq \ell_i$ for each $c \in C \setminus \{w\}$. To prevent the victory of w in (C, V_i), it is necessary that we have at least $\ell_i + 1$ votes of the form $b\ \cdots$ in V_i.

Suppose that a candidate $b \in B$ wins a subelection, say, without loss of generality, (C, V_1). Since $\mathrm{Score}(b) \leq 7$, there are at most 13 voters in V_1.

It follows that in (C, V_2), w receives at least $6k + 1 - 13 = 6k - 12$ points, while $\mathrm{Score}(p) \leq 2k + 2$, $\mathrm{Score}(d) \leq 4k + 1$, and $\mathrm{Score}(b) \leq 7$ for each $b \in B$, so w is a winner of the subelection and therefore the unique winner of the whole election.

The only possibility to prevent w's victory is that p and d are unique winners of the two first-round subelections. Without loss of

[7] This is due to Condorcet voting satisfying the so-called weak axiom of revealed preferences (see, e.g., [4, 21]).

generality, we assume that p wins in (C, V_1) and d wins in (C, V_2). Since $\mathrm{Score}_{(C,V_1)}(w) = \ell_1$, it is necessary that $N_{V_1}(p, w) = \ell_1 + 1$ and thus all votes of the form $B\ d\ w\ p$ have to be in V_2. The other way around, it is necessary that all votes of the form $B\ p\ w\ d$ are in V_1 such that $N_{V_2}(d, w) = \ell_2 + 1 > \ell_2 = \mathrm{Score}(w)$.

We have the following votes in V_1:

#	preference
x_1	$w\ p\ B\ d$
x_2	$w\ d\ B\ p$
x_3	$w\ B\ p\ d$
x_4	$B \setminus S\ p\ d\ w\ S$
k	$B\ p\ w\ d$

with $0 \le x_1 \le 2k-1$, $0 \le x_2 \le 4k-2$, $x_3 \in \{0,1\}$, $0 \le x_4 \le 3k$ and the restriction that $x_1 + x_2 + x_3 + 1 = x_4 + k$. Let $\varepsilon_b = |\{S \in \mathcal{S} \,|\, b \in S \text{ and } B \setminus S\ p\ d\ w\ S \in V_1\}|$ and $\varepsilon = \min\{\varepsilon_b \,|\, b \in B\}$. Since each b is in exactly three $S \in \mathcal{S}$, we have $0 \le \varepsilon_b \le 3$. Candidate w has a score of $x_1 + x_2 + x_3$. For p, we have $N(p, w) = x_4 + k$ and $N(p, b) = x_1 + \varepsilon_b$ for each $b \in B$. We distinguish two cases for the score of p. In both cases, we will see that that $\mathrm{Score}(p) > \mathrm{Score}(w)$ is not possible unless violating one of the conditions above.

Case 1: We have $x_4 + k < x_1 + \varepsilon$ and thus $\mathrm{Score}(p) = x_4 + k$. Since $x_4 + k = x_1 + x_2 + x_3 + 1$, we have $x_2 + x_3 + 1 < \varepsilon$. For $\varepsilon > 1$, it is necessary that $x_4 \ge 2k + 1$, since we started with a no-instance. It follows that $x_4 + k \ge 3k + 1$ and $x_1 + x_2 + x_3 + 1 \le 2k + 1$, so $x_1 + x_2 + x_3 + 1 < x_4 + k$.

Case 2: We have $x_1 + \varepsilon \le x_4 + k$ and thus $\mathrm{Score}(p) = x_1 + \varepsilon$. To prevent the victory of w, it is necessary that $x_1 + \varepsilon > x_1 + x_2 + x_3$, thus $x_2 + x_3 \in \{0, 1, 2\}$. For $x_2 + x_3 \ge 1$, we receive $\varepsilon \ge 2$ and thus $x_4 \ge 2k + 1$. It follows that $x_4 + k \ge 3k + 1$ while $x_1 + x_2 + x_3 + 1 \le 2k - 1 + 2 + 1 = 2k + 2$. For $x_2 + x_3 = 0$, we have $\varepsilon \ge 1$ and $x_4 \ge k + 1$. Therefore, $x_1 + x_2 + x_3 + 1 \le 2k$ and $x_4 + k \ge 2k + 1$.

This completes the case distinction. We have shown that, regardless of how the voters are partitioned, w is always the unique winner. It follows that (C, V, p) (respectively, (C, V, w)) is a no-instance of maximin-CCPV-TP (respectively, maximin-DCPV-TP). ❑

Next, we turn to control by partition of voters when ties eliminate. Since the proofs of Theorems 4.1 and 4.3 are pretty similar and due to space limitations, we will only sketch the latter one.

Theorem 4.3. *maximin*-CCPV-TE *and maximin*-DCPV-TE *are* NP*-complete.*

Proof. Membership of these two problems in NP once more is obvious. To show NP-hardness, we slightly modify the reduction from X3C that was given in the proof of Theorem 4.1: From our given X3C instance $(B, \mathcal{S})$, we construct an election (C, V) as in that proof, except that V now contains an additional vote of the form $w\ B\ p\ d$. The pairwise comparison between the candidates is now:

$N(\downarrow, \rightarrow)$	p	w	d	$b \in B$
p	–	$4k$	$6k+1$	$2k+2$
w	$8k-1$	–	$7k-1$	$6k+2$
d	$6k-2$	$5k$	–	$4k+1$

We claim that $(B, \mathcal{S})$ is in X3C if and only if (C, V, p) is a yes-instance of maximin-CCPV-TE (respectively, (C, V, w) is a yes-instance of maximin-DCPV-TE).

From left to right, let $(B, \mathcal{S})$ be a yes-instance of X3C. Let $\mathcal{S}' \subseteq \mathcal{S}$ with $|\mathcal{S}'| = k$ and $\bigcup_{S \in \mathcal{S}'} S = B$. Partition V into V_1 and V_2 such that V_1 contains the following votes:

#	preference	for each
$2k-1$	$w\ p\ B\ d$	
1	$B \setminus S_i\ p\ d\ w\ S_i$	$S_i \in \mathcal{S}'$
k	$B\ p\ w\ d$	

and V_2 contains all the remaining votes. We claim that p will be made the unique winner by this partition. In the first subelection, (C, V_1), we have $\mathrm{Score}(w) = N(w, p) = 2k-1$, $\mathrm{Score}(p) = N(p, b) = 2k$, $\mathrm{Score}(d) = N(d, p) = 0$, and $\mathrm{Score}(b) \le 7$. Therefore, candidate p proceeds to the final round from this subelection. In the second subelection, (C, V_2), we have $\mathrm{Score}(w) = N(w, d) = 4k$, $\mathrm{Score}(d) = N(d, b) = 4k$, $\mathrm{Score}(p) = N(p, b) = 2$, and $\mathrm{Score}(b) \le 7$. Thus d and w both win this subelection. The tie-handling rule TE blocks them both, so no one moves forward from this subelection, and p wins the final round. It follows that p is the only candidate in the final runoff and therefore the sole winner. It follows that (C, V, p) (respectively, (C, V, w)) is a yes-instance of maximin-CCPV-TE (respectively, maximin-DCPV-TE).

The direction from right to left can be shown similarly as in the proof of Theorem 4.1 and is, again, only sketched here due to space constraints. Candidate w not reaching the final runoff is equivalent to w losing in one subelection with a point difference of one to the winner and both w and another candidate winning in the other subelection so that the tie-handling rule prevents that w moves forward to the final round. It is sufficient to consider what happens if p and w win a subelection. Let $c \in \{p, d\}$ and c' be the other candidate. If c is the unique winner of the first subelection and c' is a winner of the other subelection, it follows that each vote $B\ c'\ w\ c$ is in V_2 and each vote $B\ c\ w\ c'$ is in V_1. In the following, we use ε like in the proof of Theorem 4.1. We again distinguish two cases.

Case 1: p is the unique winner in (C, V_1). In V_1 we have the following votes:

#	preference
x_1	$w\ p\ B\ d$
x_2	$w\ d\ B\ p$
x_3	$w\ B\ p\ d$
x_4	$B \setminus S_i\ p\ d\ w\ S_i$
k	$B\ p\ w\ d$

We have the restriction $x_1 + x_2 + x_3 + 1 = x_4 + k$. To ensure that p is the unique winner, it is necessary that $\mathrm{score}(w) = x_1 + x_2 + x_3 < \mathrm{score}(p) \le N(p, b) = x_1 + \varepsilon$. It follows that $x_2 + x_3 < \varepsilon$ and $\varepsilon \in \{1, 2, 3\}$. For $\varepsilon = 1$, we have $x_4 \ge k + 1$ and thus $x_4 + k \ge 2k + 1$, which is a contradiction since $x_1 \le 2k - 1$. For $\varepsilon \in \{2, 3\}$, we have $x_4 \ge 2k + 1$ and thus $x_1 + x_2 + x_3 + 1 \le x_1 + 3$ and $x_4 + 2k \ge 3k + 1$. It follows that $x_1 \ge 3k - 2$, which again is a contradiction.

Case 2: d is the unique winner in (C, V_1). The list of votes is as in Case 1, except that we have $2k$ votes $B\ d\ w\ p$ instead of the k votes $B\ p\ w\ d$. We have the restriction $x_1 + x_2 + x_3 + 1 = x_4 + 2k$. To ensure that d is the unique winner, it is necessary that $\mathrm{score}(w) = x_1 + x_2 + x_3 < \mathrm{score}(d) \le N(d, b) = x_2 + \varepsilon$. It follows that

$x_1 + x_3 < \varepsilon$. For $\varepsilon = 1$, we have $x_4 \geq k + 1$ and $x_1 = x_3 = 0$. In the other subelection, (C, V_2), we have $\text{score}(w) = 6k - 1 - x_2$ and $N(p, b) \leq 2k$. If p were winning subelection (C, V_2), we would have that $x_2 \geq 4k - 1$, which is a contradiction, though. For $\varepsilon = 2wehavex_4 \geq 2k + 1$, and for $\varepsilon = 3$ we have $x_4 = 3k$. In both cases, we receive a contradiction, since our restrictions cause that $x_2 \geq 4k - 1$ and $x_2 \geq 5k - 3$, respectively.

It follows that (C, V, p) (respectively, (C, V, w)) is a no-instance of maximin-CCPV-TE (respectively, maximin-DCPV-TE). ❑

5 Conclusions

We have completed the picture regarding the control complexity of maximin voting by solving the remaining eight open problems related to control by partition of candidates and voters for this rule. Thus, with the only exception of Schulze voting for which the complexity of destructive control by adding and deleting candidates is still open, the control complexity for (almost) all voting rules mentioned by Faliszewski and Rothe [13] in their chapter on control and bribery in the *Handbook of Computational Social Choice* have been settled now. Chapter (almost) closed.

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