

# Reasoning with Justifiable Exceptions in Contextual Hierarchies

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**Abstract.** The problem of reasoning with context dependent knowledge has recently gained interest in the area of description logic-based knowledge bases (KBs). Among the several proposals, we consider the Contextualized Knowledge Repository (CKR) framework. The CKR model has been recently extended with the capability of reasoning with global (context independent) defeasible axioms that can be overridden by local (context specific) knowledge. In CKR applications it is often useful to reason over a hierarchical organization of contexts. We highlight here our recent efforts on extending the CKR framework to allow for the representation of exception handling in the inheritance of knowledge across local contexts. We first concentrated on a limitation to a particular kind of context organization, i.e., ranked hierarchies, which allows us to simplify the definition of reasoning procedures. We then further generalized the proposal to extend the reasoning on exception handling over general contextual hierarchies. In this paper we summarize the basic definitions for simple CKRs with Justifiable Exceptions, the emerging computational properties, and the ASP-based reasoning procedures that we developed. Moreover, we highlight the open challenges in generalizing the approach and our future directions.

## 1 INTRODUCTION

In the area of Knowledge Representation and Reasoning, representing and reasoning with contexts is a well-known problem: this topic has recently gained increasing interest in the Semantic Web area, due to the need for interpreting knowledge resources with respect to contextual information given in their metadata. This led to a number of (description) logic based approaches, e.g., [5, 6, 8]. Among DL based formalisms, we consider the recent proposal of the *Contextualized Knowledge Repository (CKR) framework* [8], with its latest formulation in [1]. A CKR knowledge base is a two-layer structure where the higher level consists of a *global context* and the lower level consists of a set of *local contexts*. The global context contains context-independent knowledge about the domain (*global object knowledge*), propagated to all local contexts, and the structure of the local contexts (*meta-knowledge*). Local contexts contain facts and axioms that hold under specific situations (e.g., a specific period of time, region in space etc.). The CKR framework was extended in [1] by introducing a notion of *justifiable exceptions*. Axioms in the global context may be specified as *defeasible*, meaning that in general they are applied to instances in the local contexts, but they can be “overridden” on some exceptional instance if they would cause a local contradiction. The paper also provides a method for instance checking and conjunctive query answering in CKRs by a translation to datalog programs

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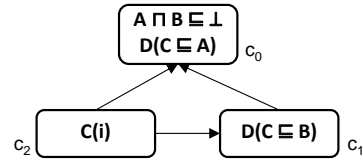


Figure 1. Example sCKR  $\mathfrak{R}_{ex}$

(under Answer Set semantics). A limitation of the proposal in [1] is that defeasible axioms can appear only in the global context and no inheritance across local contexts can be defined. In general, one may want to specify more complex structures of contexts and control the (defeasible) knowledge inheritance across them, e.g., in the common case of contexts hierarchies specified by a context *coverage* relation [8].

With this objective, in [4] we generalize the approach of [1] by allowing for local defeasible axioms and coverage contextual hierarchies. For the interpretation of overridings, we prefer models that prioritize the validity of defeasible axioms at the most specific contexts: in this way, the knowledge propagation preserves the intuitive reading where the more specific contexts refine (possibly by overriding) the knowledge coming from more general contexts. In [4] we concentrate on *ranked contextual hierarchies*, namely hierarchies that can be divided in a linear order of levels: this restriction permits to define a simple *global* model preference based on the level of the overridden axioms. Moreover, such simple model ordering allows us to easily adapt the translation to ASP programs from [1] by computing preference across answer sets by means of *weak constraints* [7]. In [2], we continue this line of work by considering the case of CKRs with *general coverage* structures. In order to manage the interpretation of overriding in general hierarchies, we need to adopt a *local* preference on models. Basically, we still prefer models which override the axioms in the higher contexts in the hierarchy: however, while in [4] preference was mapped on the linear approximation provided by levels, with general hierarchies such preference has to be defined by considering the local coverage relations of the contexts of the overridden axioms. This aspect reflects also on the ASP-based reasoning method: we formulated an algorithm, based on the semantic definition of preference, that is able to select the “preferred” answer sets which encode the expected interpretation of inheritance. In the next sections, we highlight the contributions of these two recent works and discuss the open challenges in extending our approach.

## 2 CONTRIBUTIONS

**Simple CKR with justified exceptions.** In [2, 4] we introduce a simplification of the CKR definitions from [1] in order to concentrate on the hierarchical organization of contexts. A *simple CKR (sCKR)*

is thus defined as a pair  $\mathfrak{R} = \langle \mathcal{C}, K_c \rangle$  of (i) a poset  $\mathcal{C} = (\mathbf{N}, <)$  of context names  $\mathbf{N}$  on the *coverage* relation  $<$ , and (ii) a set  $K_c$  of local context knowledge bases for each  $c \in \mathbf{N}$ . In addition to standard DL expressions, local KBs can contain *eval* expressions that allows for importing knowledge from a specific context, and *defeasible axioms*, denoted  $D(\alpha)$  for a DL axiom  $\alpha$ . In Figure 1, we depict an example sCKR  $\mathfrak{R}_{ex}$ , where arrows represent the coverage structure (e.g.,  $c_1 < c_0$ ) while the box contents represent context KBs. In [4] we restrict the form of  $\mathcal{C}$  to *ranked hierarchies*, namely hierarchies for which a linear notion of *level* can be defined. The hierarchy of  $\mathfrak{R}_{ex}$  is not ranked ( $c_1$  is reached from  $c_2$  on paths of different length); however, it becomes a two-level ranked hierarchy if we remove  $c_2 < c_1$ .

An sCKR interpretation  $\mathfrak{I}$  consists of sets  $I(c)$  of DL interpretations for each context  $c$ . The peculiarity of the semantics is how defeasible axioms are interpreted.

For each context  $c$ , interpretations are paired with a set  $\chi(c)$  of *clashing assumptions* of the form  $\langle \alpha, \mathbf{e} \rangle$  which amounts to the assumption that a defeasible axiom  $D(\alpha)$  from a higher context is not inherited by the local instance  $\mathbf{e}$  in  $c$ . Local interpretations are prevented from applying defeasible axioms on such exceptional instances. However, exceptions must be *justified* by the interpretation: for each  $\langle \alpha, \mathbf{e} \rangle$ , there needs to exist a set  $S$  of local assertions, called *clashing set*, s.t.  $S \cup \{\alpha(\mathbf{e})\}$  is locally unsatisfiable. Different justified interpretations are possible, in particular given the alternative ways of inheriting and instantiating defeasible axioms.

For example, for  $\mathfrak{R}_{ex}$  interpretations with  $\chi_1(c_2) = \{\langle C \sqsubseteq A, i \rangle\}$  and  $\chi_2(c_2) = \{\langle C \sqsubseteq B, i \rangle\}$  are both justified. We enforce the natural reading where the axioms in more specific contexts override more general axioms: for ranked hierarchies [4], we provide a *global* preference on models by comparing the level of their clashing assumptions; for general hierarchies [2] with no notion of level, a direct comparison by a *local* ordering on the clashing assumptions sets  $\chi(c)$  is defined based on the coverage: preference on models is then lifted from such ordering, by considering the models that maximize overridings to the higher contexts. In  $\mathfrak{R}_{ex}$ , this ordering prefers  $\chi_1(c_2) = \{\langle C \sqsubseteq A, i \rangle\}$ .

**Computational properties.** We studied the computational complexity of major reasoning tasks in the case of reasoning on ranked and general hierarchies. In particular, we can show that *axiom entailment* is  $\Delta_2^p$ -complete for ranked hierarchies and  $\Pi_2^p$ -complete for general hierarchies, while *CQ-answering* is  $\Pi_2^p$ -complete: thus, with respect to [1], reasoning with preferences increases the complexity of entailment, but it does not for CQ answering.

**ASP based reasoning procedures.** In [1], reasoning is provided by a translation of input CKR to datalog programs under Answer Set semantics. To adapt this ASP-based reasoning to contextual hierarchies, notably we need to encode the definition of model preference: this corresponds to an ordering over the respective answer sets of the sCKR translation. In the case of ranked hierarchies [4], this answer set ordering is obtained by *weak constraints* [7] and directly encoded in the translation. Intuitively, each overriding recognized in an answer set is associated with its level in the hierarchy. By a set of weak constraints on the presence and level of overridings, we prefer those answer sets where overridings are fewer and at higher levels. For general hierarchies [2], the complex comparison on models cannot be easily encoded inside the translation: we provided a general algorithm `PreferModels` that, taking as input all answer sets of the sCKR translation, produces the set of preferred answer sets by implementing the semantic definition of the local preference on overridings. In both cases, the correctness of the reasoning method can be proved by showing the correspondence between the preference computed on answer sets and the definition of model ordering in the semantics.

### 3 DISCUSSION AND CONCLUSION

This paper highlights the line of work started in [4] and continued in [2] on CKR frameworks with defeasible axioms in local contexts that propagate knowledge along a context hierarchy. We considered the cases of ranked and general hierarchies of contexts: CKR model preference is obtained via the notion of level of overridings in the ranked case, while in the general case from a local ordering on clashing assumptions. The ASP based reasoning method from [1] may be extended to reason on preferred models: in ranked hierarchies this can be achieved by weak constraints, while in general hierarchies by an algorithm for testing the complex condition on model preference. For extending this line of work, several challenges are identified:

**Unnamed individuals.** While in [1, 2, 4] we considered CKRs based on *SROIQ-RL* (i.e., OWL RL), we recently considered other DL languages: for example, in [3] CKR based on  $\mathcal{EL}_\perp$  were defined, which introduce reasoning on “unnamed” existential individuals. Due to the relation with the datalog translation, we only considered exceptions over “named” individuals that occur in the CKR. Enhancing the framework with reasoning about unnamed contexts would be important, but may lead to undecidability and requires restrictions.

**Preference refinement.** More “fine-grained” or alternative notions of preference on overridings are conceivable. For example, the current formulations do not compare clashing assumptions w.r.t. which defeasible axioms and instances they involve. To understand meaningful preferences and their semantic properties, and to study their expressiveness and complexity is nontrivial.

**Context relations.** Besides the coverage hierarchy, different contextual relations with custom rules for knowledge propagation can be introduced and their interaction studied. Similarly, to study the interaction with a defeasible version of the *eval* operator [1] for propagation of knowledge on specific contexts is intriguing but challenging.

**Reasoning procedure and implementation.** A final challenge concerns simplification and optimization of the reasoning methods: this also depends on the considered form of hierarchies and the definition of the preference. It is also important to identify low complexity fragments that can be evaluated efficiently. These methods can be then implemented, e.g., by extending the `CKReval` prototype [1].

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