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Split Manipulations in Cost Sharing of Minimum Cost Spanning Tree

Taiki Todo and **Makoto Yokoo**[1 2]

Abstract. This paper studies minimum cost spanning tree (MCST) problems, in which an agent can behave as multiple agents by adding fake accounts. Since such *split manipulations* may increase the cost of MCST, it is important to (i) design a cost allocation rule under which no agent has an incentive to split her accounts, and (ii) analyze the resistance of the existing cost allocation rules against split manipulations. We first show that there exists no cost allocation rule that is both efficient and split-proof under the general domain. We then focus on the MCST problems with monotonic weight functions and show that there exists a cost allocation rule that is efficient, core-selecting, and split-proof. We finally analyze the resistance of the Bird rule, one of the most studied cost allocation rules in the literature, against split manipulations from three different perspectives: the mixed price of anarchy, the computational difficulty of manipulation, and domain restrictions.

1 INTRODUCTION

In micro-economics literature, designing rules/mechanisms that achieve good properties is a critical problems (also known as *mechanism design*). In this paper we study cost allocation problems [27], where there is a set of agents and a cost function that assigns a cost to each nonempty coalition/subset of agents. The intuition is that each coalition needs to pay a corresponding cost to receive a service. Cost allocation problems arise in many real life situations, including where individuals work together, all of whom have their own purposes. For example, the cost share for building a landing slot in an airport was determined based on the well-known Shapley value [20].

Efficiency and *core-selection* are well-studied properties in cost allocation problems. A cost allocation rule is said to be efficient if for any coalition, the cost assigned by the cost function is just barely covered by the coalition. Designing an efficient cost allocation rule is crucial from the perspective of feasibility; the service cannot be provided if the cost is not covered. Also, a cost allocation rule is said to be core-selecting if no coalition of agents has an incentive to make a cartel and abandon the other agents. By definition, core-selecting cost allocation rules encourage the participation of agents.

Manipulations by splitting accounts (and their variants) have been considered in several mechanism design environments, such as auctions [30, 31], resource allocation [12, 14], matching [29], cooperative games [23, 1], and scheduling [21]. As well as such environments, in an application of cost allocation problems, agents represent groups of accounts, e.g., labor unions or nations, and therefore might have incentive to split themselves into smaller units. Such split manipulations have also been studied in cost allocation problems [17]. In this paper we analyze the effect of split manipulations in cost allocation rules that are both efficient and core-selecting.

Split-proofness can be defined as an analogy of strategy-proofness in the literature of mechanism design. A cost allocation rule is said to be *split-proof* if no agent can benefit by splitting its account, regardless of the actions of other agents. Even for a restricted class of cooperative games, there is no cost allocation rule that is both core-selecting and split-proof [17]. Thus, in this paper we further restrict the domain of problems by focusing on *minimum cost spanning tree (MCST) problems*, which are one of the most common cost allocation problems.

In an MCST problem, there is a complete graph, where each agent has a subset of nodes (say, houses) under its control, including its true node, and needs to be connected to a special node called a source. A cost allocation rule for an MCST problem determines how to share (among all participating nodes) the cost of the MCST. A split manipulation by an agent uses multiple nodes, in addition to its true node. In our model, mechanism designers can observe possible participating nodes but cannot observe who owns which nodes. Since split manipulations increase the cost of the MCST (and thus reduce social welfare), designing split-proof cost allocation rules is crucial.

There have been several cost allocation rules for MCST problems [4, 11, 15, 9, 28, 3], especially the Bird rule [4], which has attracted much attention. Among several good properties including efficiency and core-selection achieved by the Bird rule, its simplicity is perhaps its greatest advantage: for an agent i and a given minimum cost spanning tree γ, the cost of i is the weight of the first edge that i uses to access the source node in γ. From an algorithmic viewpoint, the cost of i is given as the cost of the edge that connects i to the (growing) spanning tree by Prim's algorithm [24] starting from the source node.

Our contribution: We first show that there exists no cost allocation rule that is both efficient and split-proof under the general MCST problem domain. We then focus on MCST problems with monotonic weight functions and show the existence of a cost allocation rule that is efficient, core-selecting, and split-proof. We finally analyze the effect of splitting manipulations in the Bird rule and obtain the following results: (i) the Bird rule is not split-proof even under the domain of MCST problems with monotonic weight functions; (ii) the mixed price of anarchy [18] of the Bird rule is proportional to the number of agents; (iii) determining the existence of a beneficial split manipulation is NP-complete; and (iv) the provision of a sufficient condition for the Bird rule to become split-proof.

[1] Graduate School of Information Science and Electrical Engineering, Kyushu University, Fukuoka, 819-0395 Japan, email: {todo, yokoo}@inf.kyushu-u.ac.jp
[2] RIKEN Center for Advanced Intelligence Project, Tokyo, 103-0027 Japan

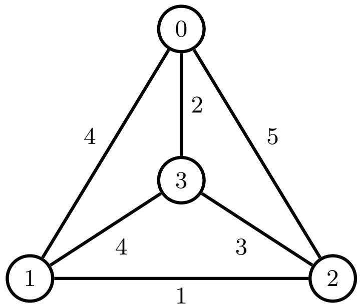

Figure 1. Example of MCST problem

2 PRELIMINARIES

Let $\mathcal{N}$ be the set of all possible *nodes*, and let $N \subseteq \mathcal{N}$ be the set of all *agents*. For a given subset of nodes $N' \subseteq \mathcal{N}$, let $G^{N'} = (N' \cup \{0\}, E^{N'})$ denote a complete undirected graph, where $\{0\}$ indicates a special node called a *source* and $E^{N'}$ indicates the set of all possible edges connecting the nodes in N'. Also, let w be a weight function that assigns a non-negative value to each edge, i.e., $w : E^{\mathcal{N}} \to \mathbb{R}_{\geq 0}$. A *minimum cost spanning tree (MCST) problem* is defined as $M = (\mathcal{N}, N, w)$, where the second argument corresponds to the set of participating nodes. For a given MCST problem $M = (\mathcal{N}, N, w)$ and a subset $N' \subseteq \mathcal{N}$ of the nodes, let $M(N')$ denote the MCST problem s.t. the participating nodes are replaced by N', i.e., $M(N') = (\mathcal{N}, N', w)$. Let $\mathcal{M}$ denote the set of all possible MCST problems.

For a given $M = (\mathcal{N}, N, w) \in \mathcal{M}$, let γ^M denote an MCST whose root is 0 for graph G^N with weight function w. It is possible that there is more than one MCST γ^M for a given M. For a given γ^M and each $i \in N$, let λ_i^M denote the parent of i in γ^M. Also, let $m(M)$ denote the cost of the MCST of G^N, i.e., $m(M)$ equals the sum of the edge weights in the MCST of G^N.

We next define the agents' possible actions. For a given $M = (\mathcal{N}, N, w) \in \mathcal{M}$, let $S = (S_i)_{i \in N}$ denote a partition of the remaining nodes, s.t. $\bigcup_{i \in N} S_i = \mathcal{N} \setminus N$, $S_i \subseteq \mathcal{N} \setminus N$ for any $i \in N$, and $S_i \cap S_j = \emptyset$ for any pair $i, j (\neq i) \in N$. Here S_i indicates the set of nodes that agent i can use. Let $\mathcal{S}^M$ be the set of all such partitions for a given $M \in \mathcal{M}$.

We now define the *cost allocation rules*. For a given $M = (\mathcal{N}, N, w) \in \mathcal{M}$, a cost allocation π for M is a tuple $(\pi_i)_{i \in N} \in \mathbb{R}_{\geq 0}^{|N|}$, where π_i indicates the cost of agent i. Also, let Π^M be the set of all cost allocations for a given $M \in \mathcal{M}$, and let $\Pi = \bigcup_{M \in \mathcal{M}} \Pi^M$. A cost allocation rule is described as a function $f : \mathcal{M} \to \Pi$ s.t. for any $M \in \mathcal{M}$, $f(M) \in \Pi^M$. For a cost allocation rule f, for a given $M = (\mathcal{N}, N, w) \in \mathcal{M}$, and for $i \in N$, let $f_i(M)$ denote the cost of agent i determined by the cost allocation rule f for the MCST problem M.

Now we are ready to formally define several properties of cost allocation rules. We define three properties: efficiency, core-selection, and split-proofness.

Definition 1. *A cost allocation rule f is* efficient *under $\mathcal{M}'(\subseteq \mathcal{M})$ if $\forall M = (\mathcal{N}, N, w) \in \mathcal{M}'$,*

$$\sum_{i \in N} f_i(M) = m(M)$$

holds.

Table 1. Split-proof cost allocation rule f

N'	f_1	f_2	f_3
$\{1,2,3\}$	2	2	2
$\{1,2\}$	5/2	5/2	
$\{1,3\}$	4		2
$\{2,3\}$		3	2

Table 2. Rule f' violating split-proofness

N'	f'_1	f'_2	f'_3
$\{1,2,3\}$	1	3	2
$\{1,2\}$	4	1	
$\{1,3\}$	4		2
$\{2,3\}$		3	2

Definition 2. *A cost allocation rule f is* core-selecting *under $\mathcal{M}'(\subseteq \mathcal{M})$ if $\forall M = (\mathcal{N}, N, w) \in \mathcal{M}'$, $\forall T \subseteq N$,*

$$\sum_{i \in T} f_i(M) \leq m(M(T))$$

holds.

Efficiency requires that the cost of any MCST is just covered enough by the participating nodes/agents. The core-selection property requires that no coalition T of agents has an incentive to form a cartel and get higher utility by abandoning all the other agents. By choosing T s.t. $|T| = 1$, we can see that, under a core-selecting cost allocation rule, each agent is incentivized to participate, i.e., core-selection implies a property called voluntary participation. In this paper, we focus on cost allocation rules that are both efficient and core-selecting.

Definition 3. *A cost allocation rule f is* split-proof *under $\mathcal{M}'(\subseteq \mathcal{M})$ if $\forall M = (\mathcal{N}, N, w) \in \mathcal{M}'$, $\forall S = (S_i)_{i \in N} \in \mathcal{S}^M$, $\forall i \in N$, $\forall S'_i \subseteq S_i$,*

$$f_i(M) \leq \sum_{i' \in i \cup S'_i} f_{i'}(M(N \cup S'_i))$$

holds.

Here the left-hand side indicates the original cost of agent i when she sincerely participates only under its true node i, and the right-hand side indicates the sum of the costs of accounts S'_i submitted by i's split manipulation. That is, split-proofness requires that no agent can be better off by splitting her account. The following example demonstrates how such a split manipulation works in MCST problems.

Example 1. *Consider an MCST problem $M = (\mathcal{N}, N, w)$ defined as follows: $\mathcal{N} = \{1, 2, 3\}$, $N = \{1, 2\}$, and w is s.t. $w(0, 1) = w(1, 3) = 4$, $w(0, 2) = 5$, $w(0, 3) = 2$, $w(1, 2) = 1$, and $w(2, 3) = 3$, where $w(i, j)$ is the weight of the edge between nodes i and j (Fig. 1).*

Consider a cost allocation rule f described in Table 1. When $S = (S_1, S_2) = (\{3\}, \emptyset)$, agent 1 cannot be better off by adding 3; her cost is originally $f_1(M) = 5/2$, and it increases to $f_1(M(N \cup$

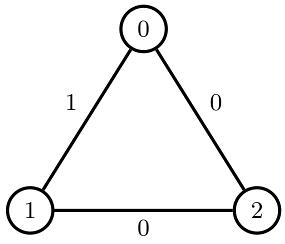

Figure 2. General incompatibility between efficiency and split-proofness

$\{3\})) + f_3(M(N \cup \{3\})) = 2 + 2 = 4$ by adding 3. Readers can easily see that no split manipulation is beneficial.

Consider another rule f', described in Table 2. In contrast to the above f, when $S = (\{3\}, \emptyset)$, agent 1 has an incentive to add 3 in f'; her cost decreases from $f'_1(M) = 4$ to $f'_1(M(N \cup \{3\})) + f'_3(M(N \cup \{3\})) = 1 + 2 = 3$ by adding 3.

3 CORE-SELECTION AND SPLIT-PROOFNESS

In this section, we investigate the (in)compatibility of split-proofness with efficiency and core-selection. We first show the incompatibility of these properties under $\mathcal{M}$, even without the property of core-selection.

Theorem 1. *There is no cost allocation rule that is efficient and split-proof under $\mathcal{M}$.*

Proof. Consider an MCST problem $M = (\mathcal{N}, N, w)$, where $\mathcal{N} = \{1, 2\}$, $N = \{1\}$, and the weight function w is s.t. $w(0, 1) = 1$ and $w(0, 2) = w(1, 2) = 0$ (see Fig. 2). It is easy to see that

$$m(M(N)) = 1, \;\; m(M(N \cup \{2\})) = 0.$$

Therefore, if a cost allocation rule is efficient, we must have both

$$f_1(M) = 1$$

and

$$f_1(M(N \cup \{2\})) = f_2(M(N \cup \{2\})) = 0.$$

Thus, for a possible partition $S = (S_1) = (\{2\})$,

$$f_1(M) > \sum_{i' \in \{1,2\}} f_{i'}(M(N \cup \{2\}))$$

holds. This means that the agent 1 has an incentive to add node 2, which violates the condition of split-proofness. □

We can easily verify the independence of the properties. A strange rule, which forces every node to pay a sufficiently large constant, regardless of the weight function, is split-proof but not efficient. On the other hand, the Bird rule is efficient but not split-proof, which will be shown in Example 3 of Section 4.

The essential point about the impossibility is that the weight function in Fig. 2 is not monotonic; adding node 2 reduces the cost of the minimum cost spanning tree from 1 to 0. However, for such a case, a mechanism designer might hesitate to prevent split manipulations, since the social cost may also decrease by split manipulations. Therefore, from the next subsection, we focus on monotonic weight functions.

Notice that the above theorem only shows the existence of an MCST problem with a non-monotonic weight function for which there exists no cost allocation rule that is efficient and split-proof. In other words, for *some* specific MCST problems with non-monotonic cost functions, it may be possible to find such a cost allocation rule with both properties.

3.1 Compatibility under Monotonic Domain

As discussed above, efficiency and split-proofness are incompatible under $\mathcal{M}$. In what follows, we focus on *monotonic weight functions*.

Definition 4. *An MCST problem $M = (\mathcal{N}, N, w) \in \mathcal{M}$ is an* MCST problem with a monotonic weight function *(MCST-M) if $\forall N'' \subset N' \subseteq \mathcal{N}$,*

$$m(M(N'')) \leq m(M(N'))$$

holds. Let $\bar{\mathcal{M}}$ denote the set (domain) of all possible MCST-Ms.

The main purpose in this section is to show the existence of a cost allocation rule that is efficient, core-selecting, and split-proof under $\bar{\mathcal{M}}$. First, we give a definition of another property called *population monotonicity* [26].

Definition 5. *A cost allocation rule f is* population monotonic *under $\mathcal{M}'(\subseteq \mathcal{M})$ if $\forall M = (\mathcal{N}, N, w) \in \mathcal{M}'$, $\forall N'' \subset N' \subseteq \mathcal{N}$, $\forall i \in N''$,*

$$f_i(M(N')) \leq f_i(M(N''))$$

holds.

That is, in a population monotonic cost allocation rule, the cost of an agent $i \in N''$ never increases after the entry of the set $N' \setminus N''$. The following theorem, which is the main contribution in this section, is essential for the existence of efficient and split-proof cost allocation rules.

Theorem 2. *Under $\bar{\mathcal{M}}$, any efficient and population monotonic cost allocation rule is core-selecting and split-proof.*

Proof. Assume that a cost allocation rule f is efficient and population monotonic. It is easy to see that population monotonicity implies core-selection, as mentioned in, e.g., Gómez-Rúa and Vidal-Puga [13]. We then show that f is also split-proof. For any $M = (\mathcal{N}, N, w) \in \bar{\mathcal{M}}$, any $S = (S_j)_{j \in N} \in \mathcal{S}^M$, any $i \in N$, and any $S'_i \subseteq S_i$,

$$f_i(M) = m(M) - \sum_{j \in N \setminus \{i\}} f_j(M)$$

holds from efficiency, and

$$m(M) \leq m(M(N \cup S'_i))$$

holds from the fact that $M \in \bar{\mathcal{M}}$. Furthermore, since $N \subseteq N \cup S'_i$,

$$f_j(M(N \cup S'_i)) \leq f_j(M)$$

holds for any $j \in N$ from population monotonicity, and thus

$$\sum_{j \in N \setminus \{i\}} f_j(M(N \cup S'_i)) \leq \sum_{j \in N \setminus \{i\}} f_j(M).$$

Also, from efficiency, it holds that

$$m(M(N \cup S'_i)) = \sum_{j \in N \cup S'_i} f_j(M(N \cup S'_i)),$$

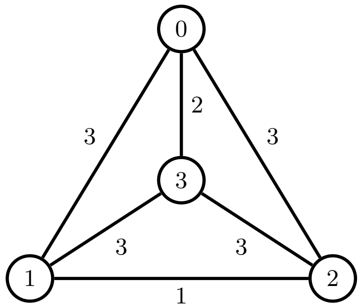

Figure 3. MCST problem $\hat{M}$, consisting of the same graph structure with Example 1, the participating nodes $N = \mathcal{N} = \{1, 2, 3\}$, and an irreducible cost function w^*

which can be written as

$$\begin{aligned} m(M(N \cup S_i')) - \textstyle\sum_{j \in N \setminus \{i\}} f_j(M(N \cup S_i')) \\ = \textstyle\sum_{i' \in \{i\} \cup S_i'} f_{i'}(M(N \cup S_i')). \end{aligned}$$

Thus,

$$\begin{aligned} f_i(M) &= m(N) - \textstyle\sum_{j \in N \setminus \{i\}} f_j(M) \\ &\leq m(M(N \cup S_i')) - \textstyle\sum_{j \in N \setminus \{i\}} f_j(M(N \cup S_i')) \\ &= \textstyle\sum_{i' \in \{i\} \cup S_i'} f_{i'}(M(N \cup S_{i'}')), \end{aligned}$$

which coincides with the definition of split-proofness. □

Since the existence of an efficient and population monotonic cost allocation rule is shown [22], the following holds directly from Proposition 2.

Corollary 1. *An efficient, core-selecting, and split-proof cost allocation rule exists under $\bar{\mathcal{M}}$.*

For example, the following cost allocation rule φ, called *folk solution*, is shown to be efficient and population monotonic [3], and thus core-selecting and split-proof; for a given $M = (\mathcal{N}, N, w) \in \mathcal{M}$ and for any $i \in N$, $\varphi_i(M)$ is the Shapley value of i in $M^* = (\mathcal{N}, N, w^*)$ with an *irreducible weight function* w^* (also known as *irreducible cost matrix*), which intuitively associates the minimum weights to edges so that $m(M) = m(M^*)$ holds and an MCST γ^{M^*} coincides with γ^M. Next we show how it works (and what the irreducible weight function looks like). For more detail, please refer to e.g., Bergantiños and Vidal-Puga [3] and Bogomolnaia and Moulin [5].

Example 2. *Consider the same graph with Example 1. For the same MCST problem $M = (\mathcal{N}, N, w)$, the irreducible weight function w^* is s.t. $w^*(0,1) = w^*(0,2) = 4$, and $w^*(1,2) = 1$. Each agent's Shapley value is $5/2$, i.e, $\varphi_1(M) = \varphi_2(M) = 5/2$. Also, for another MCST problem $\hat{M} = M(N \cup \{3\})$, the irreducible weight function $\hat{w}^*$ is s.t. $\hat{w}^*(0,1) = \hat{w}^*(0,2) = \hat{w}^*(1,3) = \hat{w}^*(2,3) = 3$, $\hat{w}^*(0,3) = 2$, and $\hat{w}^*(1,2) = 1$ (see Fig 3). Each agent's Shapley value is then 2, i.e., $\varphi_1(M(N \cup \{3\})) = \varphi_2(M(N \cup \{3\})) = \varphi_3(M(N \cup \{3\})) = 2$. Therefore, since the cost allocation by φ coincides with Table 1, it is split-proof.*

4 SPLIT MANIPULATIONS IN BIRD RULE

Although we have just seen that a core-selecting and split-proof cost allocation rule exists, we remain interested in how resistant other existing rules are to split manipulations. In particular, we analyze the Bird rule [4], whose definition is given below.

Definition 6. *The Bird rule b is the following cost allocation rule: for a given MCST problem $M = (\mathcal{N}, N, w) \in \mathcal{M}$ and any $i \in N$,*

$$b_i(M) = w(i, \lambda_i^M)$$

holds. Notice that λ_i^M is the parent of i in γ^M. When there is more than one MCST γ^M, $b_i(M)$ is defined as an average of $w(i, \lambda_i^M)$ in each γ^M.

It is known that the Bird rule is both efficient and core-selecting under $\mathcal{M}$. On the other hand, it is not split-proof even under $\bar{\mathcal{M}}$, as the following example shows:

Example 3. *Consider the MCST problem $M = (\mathcal{N}, N, w)$ described in Example 1. Since γ^M consists of the edges $(0,1)$ and $(1,2)$, $b_1(M) = 4$ holds. Also, since $\gamma^{M(N \cup \{3\})}$ consists of the edges $(0,3)$, $(3,2)$, and $(2,1)$, $b_1(M(N \cup \{3\})) = 1$ and $b_3(M(N \cup \{3\})) = 2$ hold. Therefore agent 1 has an incentive to use account 3. Actually, the cost allocation by b coincides with Table 2.*

4.1 Price of Anarchy

The issue of split manipulations is that they decrement social welfare, i.e., increase the cost of MCST. Since the Bird rule is not split-proof, it seems crucial to evaluate its worst-case performance. In this section we analyze the price of anarchy [18] of the Bird rule when split manipulations are available.

Pure strategy Nash equilibria are not guaranteed to exist. On the other hand, if we consider mixed strategies, the existence of Nash equilirbia is guaranteed. In the rest of this subsection, we consider mixed strategies. Given set S_i, let Δ_{S_i} denote the set of possible probability distributions (mixed strategies) over S_i. Also, given a mixed strategy $\delta_i' \in \Delta_{S_i}$, we write $S_i' \sim \delta_i'$ when the set $S_i' \subseteq S_i$ of accounts is realizable under the mixed strategy δ_i'. Similarly, let δ_{-i}' denote the profile of mixed strategies chosen by the agents except i, and $S_{-i}' \sim \delta_{-i}'$ denote a realizable set of accounts currently in use under δ_{-i}'.

Definition 7. *For a cost allocation rule f, for $M = (\mathcal{N}, N, w) \in \mathcal{M}$, and for $S = (S_j)_{j \in N} \in \mathcal{S}^M$, $\delta^* = (\delta_j^*)_{j \in N}$ is a (mixed-strategy)* Nash equilibrium *if for any $j \in N$, $\delta_j^* \in \Delta_{S_j}$ holds, and $\forall i \in N$, $\forall \delta_i' \in \Delta_{S_i}$,*

$$\begin{aligned} &\mathbb{E}_{S^* \sim \delta^*}\big[\sum_{i' \in S_i^* \cup \{i\}} f_{i'}(M(N \cup \bigcup_{j \in N} S_j^*))\big] \\ &\leq \mathbb{E}_{S_i' \sim \delta_i', S_{-i}^* \sim \delta_{-i}^*}\big[\sum_{i' \in S_i' \cup \{i\}} f_{i'}(M(N \cup S_i' \cup \bigcup_{j \in N \setminus \{i\}} S_j^*))\big]. \end{aligned}$$

For a cost allocation rule f, for $M = (\mathcal{N}, N, w) \in \mathcal{M}$, and for $S = (S_j)_{j \in N} \in \mathcal{S}^M$, let $\mathcal{S}^*(f, M, S)$ denote the set of all possible Nash equilibria.

Definition 8. *Given f, $M = (\mathcal{N}, N, w) \in \bar{\mathcal{M}}$, and $S = (S_j)_{j \in N} \in \mathcal{S}^M$, the* mixed price of anarchy *$P(f, M, S)$ is the minimum $\alpha \in \mathbb{R}$ s.t. $\forall \delta^* = (\delta_j^*)_{j \in N} \in \mathcal{S}^*(f, M, S)$,*

$$\mathbb{E}_{S^* \sim \delta^*}\big[m(M(N \cup \bigcup_{j \in N} S_j^*))\big] \leq \alpha \cdot m(M).$$

The lower the mixed price of anarchy of a cost allocation rule is, the higher we regard the rule (at least from the perspective of the worst-case performance). We first show the upper bound of the mixed price of anarchy, and then show that there is an MCST problem in which the ratio achieved by a pure strategy Nash equilibrium in the Bird rule matches the upper bound.

Theorem 3. *For any* $M = (\mathcal{N}, N, w) \in \bar{\mathcal{M}}$*, and any* $S = (S_j)_{j\in N} \in \mathcal{S}^M$*,*

$$P(b, M, S) \le |N'| + 1$$

holds, where $N' = \{j \mid j \in N, S_j \neq \emptyset\}$*, i.e., the set of agents with multiple accounts is denoted as* N'*.*

Proof. Let $\delta^* \in \mathcal{S}^*(b, M, S)$ be an arbitrarily chosen Nash equilibrium and, for each realization $S^* \sim \delta^*$, $N^* := N \cup \bigcup_{j\in N} S_j^*$ denote the whole set of accounts currently in use. Since the Bird rule b is efficient,

$$m(M(N^*)) = \sum_{i\in N'} \sum_{i'\in S_i^*\cup\{i\}} b_{i'}(M(N^*)) + \sum_{j\in N\setminus N'} b_j(M(N^*))$$

holds. Thus, it suffices to show that both

$$\forall i \in N', \mathbb{E}_{S^*}\big[\sum_{i'\in S_i^*\cup\{i\}} b_{i'}(M(N^*))\big] \le m(M) \qquad \text{(i)}$$

and

$$\sum_{j\in N\setminus N'} b_j(M(N^*)) \le m(M) \qquad \text{(ii)}$$

hold.

We first show the statement (i). Since δ^* is a Nash equilibrium, for any $i \in N'$, it hold that

$$\mathbb{E}_{S^*}\big[\sum_{i'\in S_i^*\cup\{i\}} b_{i'}(M(N^*))\big] \le \mathbb{E}_{S^*_{-i}}\big[b_i(M(N \cup \bigcup_{j\in N\setminus\{i\}} S_j^*))\big].$$

Also, since the Bird rule b is core-selecting,

$$\begin{aligned}\mathbb{E}_{S^*_{-i}}\big[b_i(M(N \cup \bigcup_{j\in N\setminus\{i\}} S_j^*))\big] &\le \mathbb{E}_{S^*_{-i}}\big[m(M(\{i\}))\big] \\ &= m(M(\{i\})).\end{aligned}$$

Furthermore, since M is an MCST-M problem,

$$m(M(\{i\})) \le m(M)$$

holds. Thus, we have

$$\begin{aligned}&\mathbb{E}_{S^*}\big[\textstyle\sum_{i'\in S_i^*\cup\{i\}} b_{i'}(M(N^*))\big] \\ &\le \mathbb{E}_{S^*_{-i}}\big[b_i(M(N \cup \textstyle\bigcup_{j\in N\setminus\{i\}} S_j^*))\big] \\ &\le m(M(\{i\})) \\ &\le m(M).\end{aligned}$$

We then show the statement (ii). Again, since b is core-selecting,

$$\sum_{j\in N\setminus N'} b_j(M(N^*)) \le m(M(N \setminus N'))$$

holds. Also, since M is an MCST-M problem,

$$m(M(N \setminus N')) \le m(M).$$

Thus, we have

$$\sum_{j\in N\setminus N'} b_j(M(N^*)) \le m(M(N \setminus N')) \le m(M).$$

□

From Theorem 3, the mixed price of anarchy in the Bird rule is at most $N' + 1$, where N' is the number of agents with multiple accounts. Since cost allocation rules cannot observe the exact number N' (and also the number N of agents) in our setting with anonymity, the price of anarchy is not guaranteed to be a constant, and thus, the Bird rule is not good from the viewpoint of price of anarchy. Indeed, the next theorem shows that there exists an MCST-M problem in which the ratio achieved by a pure-strategy Nash equilibrium of the Bird rule matches the upper bound.

Theorem 4. *There exists an MCST problem* $M = (\mathcal{N}, N, w) \in \bar{\mathcal{M}}$ *and a partition* $S = (S_j)_{j\in N} \in \mathcal{S}^M$ *such that*

$$P(b, M, S) = |N'| + 1$$

holds, where $N' = \{j \mid j \in N, S_j \neq \emptyset\}$*.*

Proof. Let $\varepsilon \ll 1$, k be a natural number, and $M = (\mathcal{N}, N, w)$ is s.t. $\mathcal{N} = \{1, \ldots, k, k+1, 1', \ldots, k'\}$, $N = \{1, \ldots, k+1\}$, and w is defined as follows (Fig. 4):

$$\forall i, j \in \mathcal{N} \cup \{0\}, w(i, j) = \begin{cases} (k+1)\varepsilon & \text{if } \{i, j\} \subseteq N \\ 1 - (2k+1)\varepsilon & \text{if } \{i, j\} \in \mathcal{I}' \\ 1 - j\varepsilon & \text{if } (i, j) \in \mathcal{I} \\ 1 - i\varepsilon & \text{if } (j, i) \in \mathcal{I} \\ 1 & \text{if } \{i, j\} = \{0, 1\} \\ 1 + \varepsilon & \text{otherwise,} \end{cases}$$

where $\mathcal{I}' = \{\{0, 1'\}, \{1', 2'\}, \ldots, \{(k-1)', k'\}\}$, and $\mathcal{I} = \{(1', 2), \ldots, (k', k+1)\}$. Since it is not very difficult to show that $M \in \bar{\mathcal{M}}$, the proof is omitted due to space limitations. The minumum cost spanning tree γ^M consists of the edges $(0, 1), (1, 2), \ldots, (k, k+1)$, and thus $m(M) = 1 + k(k+1)\varepsilon$ holds.

We next show that the ratio gets arbitrarily close to the bound. Let $N' = \{1, \ldots, k\}$, i.e., all the agents except $k+1$ have multiple nodes, $S = (S_1 = \{1'\}, \ldots, S_k = \{k'\}, S_{k+1} = \emptyset)$, and $M' = M(N \cup \bigcup_{j\in N'} S_j)$. Notice that $\gamma^{M'}$ consists of the edges $(0, 1'), (1', 2'), \ldots, ((k-1)', k'), (k', k+1), (k+1, k), \ldots, (2, 1)$. The action profile $(S_i)_{i\in N}$, under which every agent adds as many nodes as possible, is a (pure-strategy) Nash equilibrium, i.e., $S \in \mathcal{S}^*(b, M, S)$, because for any $i \in N'$, $\gamma^{M''}$ consists of the edges $(0, 1'), (1', 2'), \ldots, ((i-1)', j), (i, 1), \ldots, (i, i-1), (i, i+1), \ldots, (i, i+1)$ and $\sum_{i'\in S_i\cup\{i\}} b_i(M(N \cup \bigcup_{j\in N'} S_j)) = (1 - (2k+1)\varepsilon) + (k+1)\varepsilon = 1 - k\varepsilon$ and $b_i(M(N \cup \bigcup_{j\in(N'\setminus\{i\})} S_j^*)) = 1 - (i-1)\varepsilon$ hold where $M'' = M(N \cup \bigcup_{j\in(N'\setminus\{i\})} S_j^*)$. Therefore, it holds that $m(M') = k((1 - (2k+1)\varepsilon) + (k+1)\varepsilon) + (1 - k\varepsilon) = (k+1)(1 - k\varepsilon)$. Thus, we have

$$P(b, M, S) \ge \frac{m(M')}{m(M)} = \frac{(k+1)(1 - k\varepsilon)}{(1 + k(k+1)\varepsilon)},$$

which converges to $k + 1 = |N'| + 1$ for $\varepsilon \to +0$. □

Here let us explicitly show the description of those MCSTs. Assume that all the agents except i are using as many nodes as possible. If i also uses all the nodes she owns, i.e., both i and i', one of the MCST mentioned in the proof (notice that there are multiple MCSTs, but the cost of i is the same) is a path graph, such that $0 \to 1' \to 2' \to \cdots \to (k-1)' \to k' \to k+1 \to k \to \cdots \to 2 \to 1$, under which she pays the costs of the two different edges, $1 - (2k+1)\epsilon$ and $(k+1)\epsilon$. The sum is $1 - k\epsilon$.

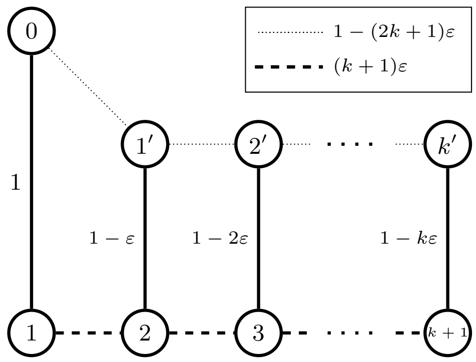

Figure 4. An MCST problem in which the price of anarchy of the Bird rule matches upper bound obtained in Theorem 3.

If she only uses her true node i, the MCST is then such that $0 \to 1' \to \cdots \to (i-1)' \to i$, i connects all the vertices in $N \cup \{(k+1)\} \setminus \{i\}$, and $(k+1) \to (k-1)' \to \cdots \to (i+1)'$. In this case, i pays the cost of the vertical edge $1-(i-1)\epsilon$, which is strictly larger than $1-k\epsilon$ for any $i < k+1$.

4.2 Complexity of Finding a Beneficial Manipulation

In practice, the computation power of each agent is limited, even though she is rational. Under such a "bounded rationality" assumption, we expect that agents will not try to solve NP-complete problems [2].

Indeed, in the Bird rule, even if a manipulator owns multiple nodes, it is not always optimal to use all of them. In Example 3, if the set $N = \{1, 2\}$ of nodes are participating agents, the partition is given as $S = (S_1, S_2) = (\{3\}, \emptyset)$, and agent 1 is a manipulator, then she can decrease her cost by adding account 3, as we have already seen; using all the nodes is optimal for this case. On the other hand, if $N = \{1\}$, $S = S_1 = \{2, 3\}$, and agent 1 is a manipulator, just using her original node 1 is optimal.

Here we show that determining whether a beneficial split manipulation exists in the Bird rule is NP-complete. We first formalize the manipulation problem in the Bird rule as follows.

Definition 9 (BENEFICIAL-SPLIT)**.**

Instance: *An MCST-M* $M = (\mathcal{N}, N, w) \in \bar{\mathcal{M}}$*, a partition* $S = (S_j)_{j\in N} \in \mathcal{S}^M$*, and an agent* $i \in N$
Question: *Does there exist a subset* $S'_i \subseteq S_i$ *of nodes such that* $b_i(M) > \sum_{i'\in S'_i\cup\{i\}} b_{i'}(M(N \cup S'_i))$ *holds?*

For instance, let $M \in \mathcal{M}$ be identical as Example 3, $S = (S_1, S_2) = (\{3\}, \emptyset)$ and $i = 1$. Agent i can decrease her cost by adding account 3. Thus, this instance of BENEFICIAL-SPLIT is an "Yes" instance.

Theorem 5. BENEFICIAL-SPLIT *is NP-complete.*

Proof. The problem is in the class NP since the cost allocation of Bird rule can be computed in polynomial time. We prove the NP-hardness by a reduction from 3-SAT. Given a 3-SAT instance 3-SAT$(C = \{c_1, \dots, c_m\}, U)$, where C denotes a collection of $m(\geq 1)$ clauses and U denotes a set of variables, and two constants, $k \in K := \{1, \dots, m\}$ and $l \in L := \{1, 2, 3\}$, let c_k^l represent the l-th literal of the k-th clause c_k, i.e., $c_k = c_k^1 \vee c_k^2 \vee c_k^3$ for any $k \in K$.

We begin by showing a transformation from a 3-SAT instance to a BENEFICIAL-SPLIT instance. Let $\varepsilon \ll 1$, $N = \{i, 1, \dots, m, m+1\}$, $N'_k = \{x_k^1, x_k^2, x_k^3\}$ for any $k \in K$, $S_i = \bigcup_{k\in K} N'_k$, and $\mathcal{N} = N \cup S_i$. Also, the weight function w is such that

$$w(p,q) = \begin{cases} \frac{1}{5m} & \text{if } \{p,q\} \in \{\{i, m+1\}\} \cup \mathcal{I} \\ 1-2\varepsilon & \text{if } \{p,q\} \in \bar{\mathcal{I}} \\ 1-\varepsilon & \text{if } \{p,q\} \in \{\{0,1\}\} \cup \mathcal{I}' \\ 1 & \text{if } \{p,q\} = \{0,i\} \\ 1+\varepsilon & \text{otherwise,} \end{cases}$$

where $\mathcal{I} = \bigcup_{k\in K}\{\{p,q\} \mid p, q \in \{k\} \cup N'_k\}$, $\bar{\mathcal{I}} = \{\{x_k^l, x_{k'}^{l'}\} \mid k, k' \in K, l, l' \in L, c_k^l = \neg c_{k'}^{l'}\}$, and $\mathcal{I}' = \bigcup_{k\in K}\{\{v, k+1\} \mid v \in N'_k\}$. Let $M = (\mathcal{N}, N, w)$ and $S = (S_i, (\emptyset)_{j\in N\setminus\{i\}})$. Since it is not very difficult to show that $M \in \bar{\mathcal{M}}$, the proof is omitted due to space limitations.

The transformed BENEFICIAL-SPLIT instance (M, S, i) appears in Fig. 5. The weight of each edge between two nodes that are not directly connected in Fig. 5 is $1 + \varepsilon$. Notice that the participating nodes in M are $N = \{i, 1, 2, \dots, m, m+1\}$. Thus, when agent i does not add an additional node from $S_i = \bigcup_{k\in K} N'_k$, it holds that $b_i(M) = 1$ because the minimum cost spanning tree γ^M contains the edge $(0, i)$.

We now show the equivalence, i.e., given any 3-SAT instance, the answer is "Yes" if and only if the answer for the transformed BENEFICIAL-SPLIT is "Yes".

Only If Part: If the instance of 3-SAT is a "Yes" instance, there exists an assignment a of the variables U s.t. all the clauses are satisfied. For given $k \in K, l \in L$, let a predicate $\rho(k, l)$ be true if and only if a literal c_k^l is true under assignment a. Let $S'_i = \{x_k^l \mid k \in K, l \in L, \rho(k,l)\}$ and $M' = M(N \cup S'_i)$. Since the assignment a satisfies all the clauses, for any $k \in K$, there exists $l \in L$ s.t. $x_k^l \in S'_i$, i.e., at least one node in N'_k is also in S'_i. Also, by definition of $\bar{\mathcal{I}}$, there exists no pair $p, q \in S'_i$ s.t. $\{p,q\} \in \bar{\mathcal{I}}$. Therefore, for any $i' \in S'_i \cup \{i\}$, $b_{i'}(M') = 1/5m$ holds, because $\gamma^{M'}$ never contains the edge $(0, i)$. Thus, $\sum_{i'\in S'_i\cup\{i\}} b_{i'}(M') = \sum_{i'\in S'_i\cup\{i\}} 1/5m \leq (3m+1)/5m < 1 = b_i(M)$ for any $m \geq 1$, which implies that the transformed instance is also a "Yes" instance.

If Part: If the transformed instance is a "Yes" instance, there exists a subset $S'_i \subseteq S_i$ s.t.

$$\sum_{i'\in S'_i\cup\{i\}} b_{i'}(M') < b_i(M) = 1, \qquad \text{(a)}$$

where $M' = M(N \cup S'_i)$. Now we show that both (a-i) for any $k \in K$, there exists $l \in L$ s.t. $x_k^l \in S'_i$, and (a-ii) there exists no pair $p, q \in S'_i$ s.t. $\{p,q\} \in \bar{\mathcal{I}}$ hold. If the statement (a-i) does not hold, then $\gamma^{M'}$ must contain the edge $(0, i)$ whose weight is 1, which contradicts the statement (a). We next assume that (a-ii) does not hold. Let $p = x_k^l$ and $q = x_{k'}^{l'} (k < k')$ s.t. $\{p,q\} \in \bar{\mathcal{I}}$. Then, $\gamma^{M'}$ contains the edge (p, q), instead of the edge $(x_{k'-1}^1, k')$, $(x_{k'-1}^2, k')$, or $(x_{k'-1}^3, k')$. Thus, $b_q(M') = 1 - 2\varepsilon$ holds. Since $b_i(M') = 1/5m$ holds, we have $\sum_{i'\in S'_i\cup\{i\}} b_{i'}(M') \geq (1-2\varepsilon) + 1/5m > 1 = b_i(M)$ for small enough ε, which contradicts the statement (a). Therefore, both (i) and (ii) must hold. Let a be the assignment of the variables of U s.t. for any $k \in K$, $l \in L$ c_k^l is true if and only if $x_k^l \in S'_i$. From (i) and (ii), a is the assignment that satisfies all the clauses. Thus, if the transformed instance is a "Yes" instance, the original instance is also a "Yes" instance. $\square$

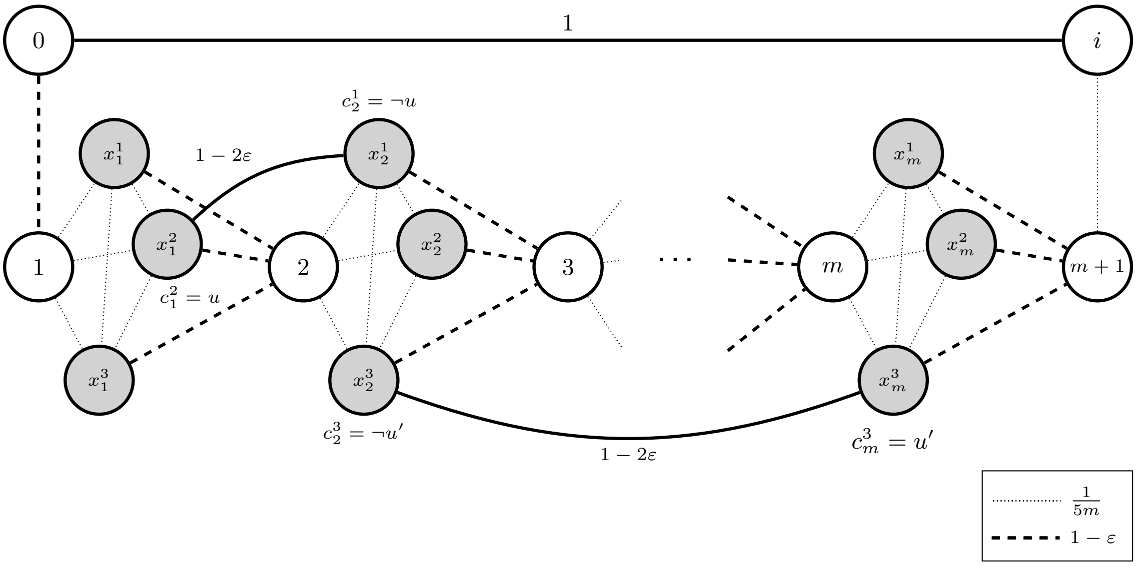

Figure 5. Transformation from 3-SAT into BENEFICIAL-SPLIT. Transformed BENEFICIAL-SPLIT is such that $N = \{i, 1, 2, \ldots, m, m+1\}$ (white nodes), and $S_i = \bigcup_{1 \le j \le m} \{x_i^1, x_i^2, x_i^3\}$ (grayed nodes).

Theorem 5 implies that in the worst case, an agent must solve a computationally difficult problem to find a beneficial split manipulation. In this sense, the Bird rule has a reasonable level of resistance against split manipulations.

As several recent papers have argued [10], discussing the computational complexity of manipulation problems is not always sufficient. Nevertheless, we believe that such a computational complexity approach is a certain first step for understanding the effect of split manipulations in MCST problems.

4.3 Split-proof Domain

It remains worthwhile to clarify under which condition on weight functions the Bird rule becomes split-proof. In this section, we provide a very intuitive sufficient condition.

For $M = (\mathcal{N}, N, w) \in \mathcal{M}$, let $\mathrm{anc}(M, i, j)$ denote the predicate that $i \in N$ is an ancestor of $j \in N$ in γ^M, and let Σ^M denote the set of all possible permutations of the nodes, i.e., $\sigma : \mathcal{N} \rightarrowtail\!\!\!\!\twoheadrightarrow \{1, \ldots, |\mathcal{N}|\}$, where $\rightarrowtail\!\!\!\!\twoheadrightarrow$ indicates a bijection.

Theorem 6. *The Bird rule b is split-proof under a domain $\mathcal{M}' \subseteq \bar{\mathcal{M}}$ of MCST-M problems if $\forall M = (\mathcal{N}, N, w) \in \mathcal{M}'$,*

1. *γ^M is unique, and*
2. *there exists a permutation $\sigma \in \Sigma^M$ s.t. $\forall N' \subseteq \mathcal{N}$ and $\forall i, j \in N'$,*

$$\mathrm{anc}(M(N'), i, j) \Rightarrow \sigma(i) < \sigma(j).$$

Proof. For the sake of contradiction, we assume that b is not split-proof under $\mathcal{M}' \subseteq \bar{\mathcal{M}}$. Then it suffices to show that there exists $M = (\mathcal{N}, N, w) \in \mathcal{M}'$ such that either γ^M is not unique, or $\forall \sigma \in \Sigma^M$, $\exists N' \subseteq \mathcal{N}$, $\exists i, j \in N'$, it holds that

$$(\sigma(i) \ge \sigma(j)) \wedge \mathrm{anc}(M(N'), i, j).$$

Since b is not split-proof, $\exists M = (\mathcal{N}, N, w) \in \mathcal{M}'$, $\exists S = (S_j)_{j \in N} \in \mathcal{S}^M$, $\exists i \in N$, $\exists S_i' \subseteq S_i$,

$$b_i(M) > \sum_{i' \in S_i' \cup \{i\}} b_{i'}(M'),$$

where $M' = M(N \cup S_i')$. Let i^* be such an agent $i \in N$. If γ^M is not unique, the proof is completed. We then consider the cases where γ^M is unique and show that $\exists j \in N$, $\mathrm{anc}(M, j, i^*)$ and $\mathrm{anc}(M', i^*, j)$.

Let $D = \{j \in N \mid \mathrm{anc}(M, i^*, j)\}$. For the sake of contradiction, we assume that for any $i' \in S_{i^*}' \cup \{i^*\}$, it holds that

$$\lambda_{i'}^{M'} \in (N \cup S_{i^*}') \setminus D.$$

Let ν be the nearest node to i^* in $\{j \in N \mid \mathrm{anc}(M', j, i^*)\}$. Since $b_{i^*}(M) > \sum_{i' \in S_{i^*}' \cup \{i^*\}} b_{i'}(M')$ holds, we have $\sum_{i' \in Q} b_{i'}(M') \le b_{i^*}(M) = w(i^*, \lambda_{i^*}^M)$, where Q is the set of nodes owned by i^* in the path from i^* to ν (note that ν is not in Q). Since $M \in \bar{\mathcal{M}}$ and γ^M is unique, $w(i^*, \nu) \le \sum_{i' \in Q} b_{i^*}(M) \le w(i^*, \lambda_{i^*}^M)$ holds. Here, $\lambda_{i^*}^M \neq \nu$ implies $\mathrm{anc}(M, i^*, \nu)$, which contradicts the definition of ν. Thus, there exists $i' \in S_{i^*}' \cup \{i^*\}$ s.t. $\lambda_{i'}^{M'} \in D$ holds, i.e., $\exists j \in N$ s.t. $\mathrm{anc}(M, j, i^*)$ and $\mathrm{anc}(M', i^*, j)$. ☐

The intuition of the proof of Theorem 6 is that, although both i^* and ν are in both M and M' (since ν is not owned by i^*), their order differs in these MCSTs, i.e., $\mathrm{anc}(M, i^*, \nu)$ and $\mathrm{anc}(M', \nu, i^*)$. This is because, since i^* can benefit from the split manipulation that connects the node i^* to ν under M', she is not connected to ν (in other words, ν must be a descendant of i^* in M) without manipulation.

The two conditions in Theorem 6 intuitively require that for any pair of two participating nodes, their ancestor-descendant relationship never switches for any set of participating nodes including themselves. For instance, in Example 3, the relationship between nodes 1 and 2 switches before and after node 3 joins.

When do the conditions hold in MCST problems? A very naive example is building a service-providing network, e.g., electricity or water, in a city where two main streets cross at a right angle, all the houses are located along one of them, and the service provider is located near the intersection. When the weight function is given as an Euclidean distance function, we can easily imagine that no split manipulation is beneficial.

5 CONCLUDING REMARKS

In this paper we analyzed the effect of split manipulations in the MCST problems and showed that no rule is both efficient and split-proof under $\mathcal{M}$. We further showed that under $\bar{\mathcal{M}}$, there exists a rule that is efficient, core-selecting, and split-proof. Concerning the resistance of the Bird rule against split manipulations, we showed that it is not split-proof, its mixed price of anarchy is proportional to the number of agents, and finding an optimal manipulation is NP-complete. We also provided a sufficient condition for the Bird rule to be split-proof.

One future work will characterize the domain of MCST problems under which an efficient, core-selecting, and split-proof rule exists. In this paper we already identified a domain of MCST problems under which the Bird rule is split-proof, but it is not tight. It might also be interesting to analyze the convergence/diffusion of the agents' selfish behavior when the market size grows, e.g., the number of nodes in the MCST problem. Considering monotonic MCST problems (mMCSTs), in which one can use non-participating nodes to minimize the cost of the spanning tree, and analyzing the existing rules by recently proposed measures such as *incentive ratio* [6, 7, 8], are other possible directions. Finally, combining MCST problems with the framework of mechanism design with information diffusion [19, 25, 16] will also be interesting; their definition of strategy-proofness requires that hiring more buyers into the market be a dominant strategy, which also looks meaningful in some application domains of cost sharing.

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