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Evaluation of Analogical Arguments by Choquet Integral

Leila Amgoud [1]

Abstract. Analogical arguments are a special type of inductive arguments, whereby perceived similarities are used as a basis to infer some further similarity that has yet to be observed. Although they are not deductively valid, they may yield conclusions that are very probably true, and may be more cogent than others in persuasive contexts.

This paper tackles the question of their evaluation. It starts by discussing their features, how they can be attacked/supported, and key considerations for their evaluation. It argues in particular for the need of semantics that are able to take into account possible interactions (synergies, redundancies) between attackers (respectively supporters) of any analogical argument. It presents principles that serve as guidelines for choosing candidate semantics. Then, it shows that existing (extension, gradual, ranking) semantics are not suitable as they may lead to inaccurate assessments. Finally, it redefines three existing semantics using the well-known Choquet Integral for aggregating attackers/supporter, and discusses their properties.

1 INTRODUCTION

Analogical reasoning is one of the most common methods by which human beings attempt to understand the world and make decisions [6, 21, 25]. It consists of exploring parallels between situations. Indeed, to give an analogy is to claim that two distinct items are alike or similar in some respect, or share features. For instance, how a doctor diagnoses diseases is like how a detective investigates crimes; the structure of an atom is like a solar system. *Analogical arguments* or arguments by analogy are a special type of inductive arguments that rely on analogies for drawing conclusions. They cite accepted similarities between two items in support of the conclusion that some further similarity exists. They have the following general schema:

Items I_1 and I_2 are similar in having properties $P_1, \ldots, P_n$.
I_1 has property Q.
Therefore, I_2 also has property Q.

An example of analogical argument is: This novel is similar to the one I have read recently; they both have a green cover (P_1). The previous book was boring (Q), then this novel will be boring as well. Staring from the common feature (P_1) for the two books, the argument concludes that they will share also the feature (Q).

Such arguments are not deductively valid, however some of them may yield conclusions that are very probable. They may also be more cogent than others in persuasive context [28]. The reason is that analogy facilitates understanding, which is crucial for persuading an audience. Consequently, several scholars mainly philosophers [13, 14, 15, 26, 27, 29] have investigated analogical arguments and have focused on their types, features, and strengths.

[1] Centre National de la Recherche Scientifique – IRIT – ANITI, France, email: amgoud@irit.fr

Despite a steady progress by philosophers in understanding how analogical arguments can be evaluated, to the best of our knowledge there is almost no study from the computational argumentation side on the kind of semantics that are suitable for this type of arguments. Two notable exceptions are the works in [8, 23], where the authors used extension semantics that have been initiated in [12].

This paper investigates from a formal point of view the type of framework that is necessary for reasoning with analogical arguments, and the family of semantics that are suitable for evaluating them.

We start by arguing that bipolarity is inevitable as any analogical argument may be supported by arguments and attacked by others. A supporter provides additional features that are shared by the items compared by the analogical argument, while an attacker highlights non-shared ones (hence cases of dissimilarity between the items). Both (supporters and attackers) are crucial for the evaluation of the strength of the targeted analogical argument. The idea is that the more features are shared and the fewer dissimilarities are highlighted between the items, the stronger the analogy between the items, and thus the stronger the analogical argument. This idea is captured by a principle, called *Monotony*, that any semantics should satisfy. A second important principle for a semantics is the so-called *Diversity* by philosophers. It states that a semantics should take into account possible interactions (synergies and redundancies) among features, and thus among respectively attackers and supporters.

We show that existing extension semantics [12] violate both principles and are thus not suitable in the context of analogical arguments. Some gradual semantics like Card-based and Weighted h-Categorizer from [5] satisfy Monotony but violate Diversity leading thus to inaccurate assessments. We argue that for satisfying Diversity, attackers (respectively supporters) should be aggregated by a function that is able to deal properly with possible interactions (synergies, redundancies) between arguments. Choquet integral [11] is one of such functions. We redefine then three existing semantics (Weighted h-Categorizer [5], Aggregation-based [3], Euler-based [4]) using Choquet Integral and show that they satisfy the desirable principles.

The paper is structured as follows: Section 2 presents a running example, Section 3 presents the formal framework for reasoning with analogical arguments, Section 4 discusses different desirable properties for their evaluation, Section 5 proposes three novel semantics and investigates their properties, Section 6 is devoted to related work, and Section 7 concludes.

2 ILLUSTRATIVE EXAMPLE

Let us illustrate the different characteristics of analogical arguments by the following simple example. Consider the six arguments below exchanged during a dialogue between friends:

A This novel has a similar plot (p) like the one we have read. The latter was boring (b), therefore the novel will be boring as well.

B The two books are alike in that their covers are both green (g).

C Their covers have both a picture of a dove (d).

D The two novels are written by the same author (a).

E Very few of the first book have been sold (s) while the new book is a hit in bookstores.

F The two books are published by different editors (e).

The argument A follows the schema presented above. It compares two items (books in this case) and claims a first similarity between them: having same plot ($P_1 = p$). Then, it concludes that two books will share also the property of being boring ($Q = b$).

The three arguments B, C, D have the following schema:

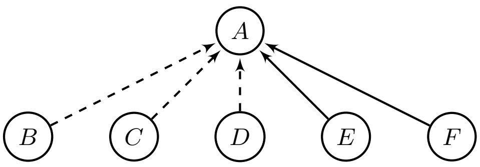
Items I_1 and I_2 have both properties $P_1, \ldots, P_n$.
*Therefore, I_1 and I_2 are **similar**.*

They sustain the similarity claimed by A by providing other common features of the two books. They are thus *supporting* A.

The two arguments E, F have the following schema:

Item I_1 has properties $P_1, \ldots, P_n$ while I_2 not.
*Therefore, I_1 and I_2 are **dissimilar**.*

They highlight features on which the two books are different. In E the feature is the number of sales and in F it is the editor. Both arguments undermine the first premise of A, which states that the two books are similar. They are thus clearly *attacking* A. The graph below depicts the relations between the arguments, where dashed lines are support relations while plain ones denote attacks.



Characteristics 1 *Reasoning about analogical arguments requires a bipolar framework (i.e., with both attack and support relations).*

In analogical reasoning, the features that are used for comparing items may not be of equal *importance*. For instance, cover's colour may be less important than book's topic. Furthermore, some groups of features may be more important than others. For instance, sharing the two features (green covers with a picture of a dove) is less important than sharing the two features (green cover and author). The two first ones are on the same topic (book's cover) while the second ones are on varied aspects: book's cover and book's author. The importance of a group of features may be more important than the sum of importances of its elements when *synergies* hold between features, and may be less than the sum of importances of features when there are *redundancies* among them.

Characteristics 2 *Reasoning about analogical arguments requires capturing properly the diversity and the interactions of features that are shared by items. This amounts to considering importance degrees of groups of features.*

The importance of (groups of) features is inherited by analogical arguments promoting them. Indeed, every argument has a *basic weight* reflecting the importance of the features it promotes. For instance, the basic weight of the argument A is the importance of the feature p, and the basic weight of B is the importance of g.

Characteristics 3 *Every analogical argument has a basic weight, which is related to the importance of the features it promotes.*

Consider now the three arguments A_1, A_2, A_3 of Figure 1. The first one is supported by two arguments promoting the features g and d, which are both on book's cover. The second argument is supported by two arguments promoting g and a (on cover and author) and the third argument is supported by arguments referring to d, a (also on cover and author). Intuitively, $\{B, C\}$ should have less impact on A_1 than $\{B, D\}$ on A_2 and than $\{C, D\}$ on A_3 due to the importance of the groups of features underlying the arguments. Thus, both A_2 and A_3 should be stronger than A_1. This shows that the possible interactions between features lead to interactions between arguments referring to them.

Characteristics 4 *Reasoning about analogical arguments requires capturing interactions between attackers/supporters. This amounts to identifying basic weights of groups of arguments.*

We have seen that features have weights reflecting their importance for comparing items (eg., books). However, the fact that a feature is important does not mean that it is necessarily relevant to an argument's conclusion. Let us consider the argument F, which claims that the two books are dissimilar because they have different editors (e). The feature e is irrelevant for concluding that a book will be boring, hence the attack from F to A is quite weak. Consider now the two following arguments:

G These two books are similar, they have the same cover. The first one has a lot of typos, therefore the second one too.

H They have been edited by the same editor.

Clearly, H supports G and this relation is relevant since one would think that editors should check books before publication. Thus, the feature "same editor" is relevant to the conclusion "having typos".

Characteristics 5 *Attack/Support relations have relevance degrees.*

3 ARGUMENTATION FRAMEWORK

This section formalizes the different characteristics discussed previously. Throughout the paper, we assume a finite set $\mathcal{F} = \{f_1, \ldots, f_i\}$ of features for comparing objects. Each feature and each subset of features has an importance degree, capturing thus the second characteristic. Such degrees are ascribed by a *capacity*, called also *fuzzy measure* by Choquet in [11], which is a function that assigns to every subset of features a value from the unit interval $[0, 1]$.

Definition 1 (Capacity) *A* capacity *over a set $\mathcal{X}$ is a function $\mathcal{V}$ from $\mathcal{P}(\mathcal{X})^2$ to $[0, 1]$ satisfying the following conditions:*

- $\mathcal{V}(\emptyset) = 0$ *(Boundary condition)*
- $\mathcal{V}(A) \leq \mathcal{V}(B)$ *whenever* $A \subseteq B \subseteq \mathcal{X}$ *(Monotonicity)*

Monotonicity means: the bigger a set, the more important it is.

We are now ready to introduce the notion of *theory*.

Definition 2 (Theory) *A* theory *is a pair $\langle \mathcal{F}, \pi \rangle$ where $\mathcal{F}$ is a finite set of features and π is a capacity over $\mathcal{F}$.*

[2] $\mathcal{P}(\mathcal{X})$ denotes the power set of the set $\mathcal{X}$.

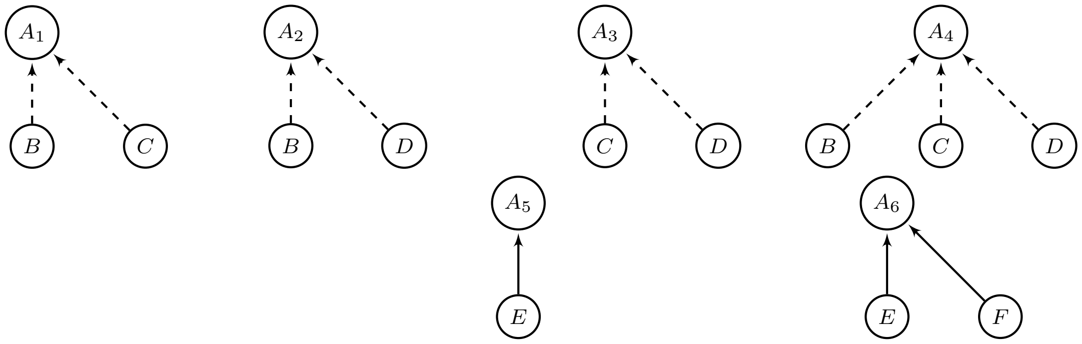


Figure 1. Examples of Graphs: Every A_i is the argument A

It is worth mentioning that the definition of capacity covers three types of capacity functions over the set of features: For $A \subseteq B \subseteq \mathcal{F}$,

Concave: $\pi(A \cup B) + \pi(A \cap B) \leq \pi(A) + \pi(B)$
Convexe: $\pi(A \cup B) + \pi(A \cap B) \geq \pi(A) + \pi(B)$
Additive: $\pi(A \cup B) + \pi(A \cap B) = \pi(A) + \pi(B)$

The first type expresses existence of redundancies between the sets A and B. The second type expresses positive synergies among the two sets. Finally, the third type expresses independence of the sets.

The diversity condition (mentioned in characteristic 2) may not be suitable in all cases. For instance, claiming that two books are similar in that they are written by the same author (f_1) is more important than claiming that they are similar in having the same colour of their covers (f_2) and are edited in the same country (f_3). Indeed, the group of features $\{f_2, f_3\}$ may be less important than the singleton $\{f_1\}$, i.e., $\pi(\{f_2, f_3\}) \leq \pi(\{f_1\})$.

Throughout the paper, we assume a finite set of analogical arguments. Every argument promotes one or more features in support of a similarity/dissimilarity between two items. An attacker of an argument highlights some features that two items do not share while a supporter shows some additional similarities between the items. This bipolarity of the framework answers characteristic 1. For meeting characteristics 3 and 4, we assume that every argument and every group of arguments has a basic weight, which is equal to the importance degree of the group of features that are considered. Finally, for meeting the last characteristic we assume that attack/support relations are weighted.

Definition 3 (Argumentation Framework (AF)) *An* AF *built on theory* $\langle \mathcal{F}, \pi \rangle$ *is an ordered tuple* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$*, where:*

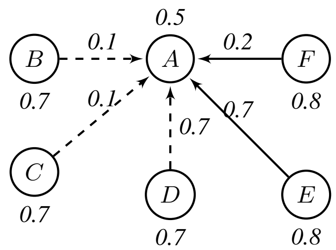
- $\mathcal{A}$ *is a non-empty finite set of analogical arguments*
- $v : \mathcal{A} \to \mathcal{P}(\mathcal{F}) \setminus \emptyset$
- σ*:* $\mathcal{P}(\mathcal{A}) \to [0, 1]$ *such that* $\sigma(\emptyset) = 0$ *and for every* $X \subseteq \mathcal{A}$*,*

$$\sigma(X) = \pi(\bigcup_{a \in X} v(a))$$

- $\mathcal{R}_s \subseteq \mathcal{A} \times \mathcal{A}$ *is a support relation*
- $\mathcal{R}_a \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation*
- $\delta : \mathcal{R}_a \cup \mathcal{R}_s \to [0, 1]$

For $a \in \mathcal{A}$, $v(a)$ is the set of features promoted by a, for $X \subseteq \mathcal{A}$ $\sigma(X)$ denotes the importance degree of X, and it is equal to the importance of the set of features promoted by the elements of X.

Example 1 *Let us now come back to our running example. The figure below shows the relevance degrees of the (attack, support) relations as well as the importance degrees of the individual arguments. In addition, we assume the following:* $\sigma(\{B, C\}) = 0.75$*,* $\sigma(\{B, D\}) = 0.9$*,* $\sigma(\{C, D\}) = 0.9$*,* $\sigma(\{B, C, D\}) = 0.9$*,* $\sigma(\{E, F\}) = 0.8$*. Note that the importance of the group* $\{E, F\}$ *is the same as the importance of any of its elements. This means that* E *and* F *are redundant and thus only one of them can be considered. The same holds for the group* $\{B, C, D\}$*. Indeed, it is as important as any of* $\{B, D\}$ *and* $\{C, D\}$*. This means that one should discard either* B *or* C *but not both.*



Property 1 *Let* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$ *be an AF. For any* $A \subseteq B \subseteq \mathcal{A}$*, it holds* $\sigma(A) \leq \sigma(B)$*. Furthermore,*

$$\sigma(\mathcal{A}) = 1 \textit{ iff } \bigcup_{a \in \mathcal{A}} v(a) = \mathcal{F}.$$

The above result shows that σ is a capacity. Moreover, it is *normalized* ($\sigma(\mathcal{A}) = 1$) when the capacity on $\mathcal{F}$ is normalized ($\sigma(\mathcal{F}) = 1$) and items are compared with respect to all features in $\mathcal{F}$.

4 EVALUATION OF ANALOGICAL ARGUMENTS

The different studies made by philosophers (eg., [13, 14, 15, 26, 27, 29]) claim that an analogical argument has a *gradual strength*, which may range from very weak to very strong depending on the following four considerations:

Relevance (Rel): The idea is to check the relevance of the features in which the items are similar to an argument's conclusion. In the example, the colour of book's cover is not relevant for concluding that a book will be boring. In other words, we need to ensure that having features $P_1, \ldots, P_n$ increases the probability of an item having feature Q. The P_i are either those features stated in the analogical argument under study or in its supporters.

Number (Num): An analogical argument is stronger when the compared items share a lot of relevant features. For instance, in Figure 1 our confidence in the conclusion of the argument A_4 is greater than in the conclusions of A_1, A_2, A_3 since the similarity between the two books is based on more common features in A_4.

Diversity (Div): To ensure that the shared features are not all of the same kind. This condition depends on the importance of groups of features. Assume three features f_1, f_2, f_3 such that $\pi(\{f_1\}) > \pi(\{f_2, f_3\})$. Intuitively, an analogical argument promoting the feature f_1 would be stronger than anyone that promotes f_2, f_3.

Disanalogy (Dis): Even if two items are similar in lots of relevant aspects, we should also consider whether there are dissimilarities between them which might cast doubt on the conclusion.

Evaluation of arguments has largely been investigated in the computational argumentation literature. Three families of semantics have been proposed [1]: *extension* semantics [12], *gradual or weighting* semantics [9], and *ranking* semantics [2].

Extension semantics look for sets of arguments, called extensions, that defend their elements against all attacks. Each extension represents an alternative set of acceptable arguments. These semantics have the following characteristics, which impede their use for the evaluation of analogical arguments. First, the effect of an attack is binary. Indeed, an attack either leads to full rejection of arguments or has no effect. Consider the argument A_5 (which is A in the running example) in Figure 1. As soon as E has a greater basic weight than A_5, all existing extension semantics will reject A_5. However, in case of analogical arguments, it is rare to have non-attacked arguments since items always differ in at least one feature.
Second, the impact of one attack is the same as n attacks. In Figure 1, being attacked by E only or by both E and F has the same effect, violating thus the above consideration (**Num**).
Third, in the bipolar case (eg., [10, 19, 7, 18, 20]), if the attack relation is empty, all arguments will be accepted. Hence, in Figure 1, the four arguments $A_1, \ldots, A_4$ are all equally acceptable, violating again the consideration (**Num**). Note that A_4 is based on a similarity which involves more features (p, g, d, a) than the others, thus A_4 should be stronger than A_1, A_2, A_3.
Finally, when both types of relations are available in a graph, the effect of attacks may be lethal. The argument A will be rejected by all existing bipolar extension semantics. Thus, as said above most analogical arguments will be rejected since they are all attackable.

Ranking semantics return for every argumentation graph a total preorder on the set of arguments. Indeed, they rank order arguments from the strongest to the weakest ones. Such semantics also are not suitable for analogical arguments. In our running example, they rank order the arguments A, B, C, D, E, F without specifying whether A is strong or not. Consider for instance the case of two distinct analogical arguments X, Y such that X attacks itself and Y is not attacked. Existing semantics would return the following: X is at least as strong as X and Y is at least as strong as Y. They do not declare that X is stronger than Y, and that Y is very weak.

Gradual semantics do not focus on acceptance/rejection of arguments, but rather on their strengths. Thus, they are more appropriate for our purpose. In what follows we will use such semantics.

Definition 4 (Semantics) *A* semantics *is a function* $\mathbf{S}$ *transforming any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$ *into a weighting* $\mathtt{s}^{\mathbf{S}}_{\mathbf{A}} : \mathcal{A} \to [0, 1]$. *Let* $a \in \mathcal{A}$, $\mathtt{s}^{\mathbf{S}}_{\mathbf{A}}(a)$ *denotes the* strength *of* a.

Remark: When the semantics and the AF are clear from the context, we write $\mathtt{s}(a)$ for short for denoting the strength of a.

This definition is very general and does not constrain the choices of semantics. In what follows, we propose some principles that a reasonable semantics for analogical arguments should satisfy. The principles will capture the considerations (**Rel, Num, Dis**). Before presenting them formally, let us first introduce some useful notations.

Notations: Let $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$ be an AF and $a \in \mathcal{A}$. We denote by $\mathtt{Att}(a)$ the set of all attackers of a in $\mathbf{A}$ (i.e., $\mathtt{Att}(a) = \{b \in \mathcal{A} \mid b\mathcal{R}_a a\}$), $\mathtt{sAtt}(a) = \{b \in \mathtt{Att}(a) \mid \mathtt{s}(b) > 0\}$, by $\mathtt{Supp}(a)$ the set of all supporters of a (i.e., $\mathtt{Supp}(a) = \{b \in \mathcal{A} \mid b\mathcal{R}_s a\}$) and $\mathtt{sSupp}(a) = \{b \in \mathtt{Supp}(a) \mid \mathtt{s}(b) > 0\}$.

Relevance (**Rel**) is captured by two principles. The first one ensures that weights of relations between arguments are properly taken into account. The basis idea is that the weaker a relation (hence the weaker the degree of relevance of the features promoted by the source to the conclusion of the target argument), the less impact it has on an argument.

Principle 1 *(Relevance) A semantics* $\mathbf{S}$ *satisfies* relevance *iff for every AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$*, for all* $X, Y \subseteq \mathcal{A}$*, for all* $a, b \in \mathcal{A}$*, for all* $x, x', y, y' \in \mathcal{A} \setminus (X \cup Y)$ *such that*

- $\sigma(\{a\}) = \sigma(\{b\}) > 0$,
- $\mathtt{Att}(a) = X \cup \{x\}$ *and* $\mathtt{Att}(b) = X \cup \{y\}$,
- $\mathtt{Supp}(a) = Y \cup \{x'\}$ *and* $\mathtt{Supp}(b) = Y \cup \{y'\}$,
- $\mathtt{s}(x) = \mathtt{s}(y)$ *and* $\mathtt{s}(x') = \mathtt{s}(y')$,
- $\forall X' \subseteq X$, $\sigma(X' \cup \{x\}) = \sigma(X' \cup \{y\})$ *and* $\forall Y' \subseteq Y$, $\sigma(Y' \cup \{x'\}) = \sigma(Y' \cup \{y'\})$
- $\forall z \in X$, $\delta((z, a)) = \delta((z, b))$, $\forall z' \in Y$, $\delta((z', a)) = \delta((z', b))$, $\delta((y, b)) \geq \delta((x, a)$ *and* $\delta((x', a)) \geq \delta((y', b))$,

the following holds:

- $\mathtt{s}(a) \geq \mathtt{s}(b)$; ***(Relevance)***
- *if* $(\mathtt{s}(a) > 0$ *and* $\delta((y, b)) > \delta((x, a)))$ *or* $(\mathtt{s}(b) < 1$ *and* $\delta((x', a)) > \delta((y', b)))$*, then* $\mathtt{s}(a) > \mathtt{s}(b)$. ***(Strict Relevance)***

The second principle is a generalization of Reinforcement proposed in [4]. It states that any argument becomes stronger if the quality of its attackers is reduced and the quality of its supporters is increased. This principle takes into account the importance of groups of features that are used for showing similarities/dissimilarities.

Principle 2 *(Reinforcement) A semantics* $\mathbf{S}$ *satisfies* reinforcement *iff, for every AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$*, for all* $X, Y \subseteq \mathcal{A}$*, for all* $a, b \in \mathcal{A}$*, for all* $x, x', y, y' \in \mathcal{A} \setminus (X \cup Y)$ *such that*

- $\sigma(\{a\}) = \sigma(\{b\}) > 0$,
- $\mathtt{Att}(a) = X \cup \{x\}$ *and* $\mathtt{Att}(b) = X \cup \{y\}$,
- $\mathtt{Supp}(a) = Y \cup \{x'\}$ *and* $\mathtt{Supp}(b) = Y \cup \{y'\}$,
- $\mathtt{s}(x) \leq \mathtt{s}(y)$ *and* $\mathtt{s}(x') \geq \mathtt{s}(y')$,
- $\forall z \in X$, $\delta((z, a)) = \delta((z, b))$, $\forall z' \in Y$, $\delta((z', a)) = \delta((z', b))$, $\delta((x, a)) = \delta((y, b))$ *and* $\delta((x', a)) = \delta((y', b))$,
- $\forall X' \subseteq X$, $\sigma(X' \cup \{x\}) = \sigma(X' \cup \{y\})$ *and* $\forall Y' \subseteq Y$, $\sigma(Y' \cup \{x'\}) = \sigma(Y' \cup \{y'\})$

the following holds:

- $\mathtt{s}(a) \geq \mathtt{s}(b)$; ***(Reinforcement)***
- *if* $(\mathtt{s}(a) > 0$ *and* $\mathtt{s}(x) < \mathtt{s}(y))$ *or* $(\mathtt{s}(b) < 1$ *and* $\mathtt{s}(x') > \mathtt{s}(y'))$*, then* $\mathtt{s}(a) > \mathtt{s}(b)$. ***(Strict Reinforcement)***

The consideration (**Num**) is captured by the following monotony principle proposed in [4]. It states that an argument is all the stronger when it is less attacked and more supported. In other words, an analogical argument is all the stronger when the items it compares share more features and have less dissimilarities

Principle 3 *(Monotony) A semantics* $\mathbf{S}$ *satisfies* monotony *iff, for any* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$*, for all* $a, b \in \mathcal{A}$ *such that:*

- $\sigma(\{a\}) = \sigma(\{b\})$,
- $\mathtt{Att}(a) \subseteq \mathtt{Att}(b)$ *and* $\forall x \in \mathtt{Att}(a)$, $\delta((x,a)) = \delta((x,b))$,
- $\mathtt{Supp}(b) \subseteq \mathtt{Supp}(a)$ *and* $\forall x \in \mathtt{Supp}(b)$, $\delta((x,a)) = \delta((x,b))$,

the following holds:

- $\mathtt{s}(a) \geq \mathtt{s}(b)$; ***(Monotony)***
- *if* $(\mathtt{s}(a) > 0$ *and* $\mathtt{sAtt}(a) \subset \mathtt{sAtt}(b))$ *or* $(\mathtt{s}(b) < 1$ *and* $\mathtt{sSupp}(b) \subset \mathtt{sSupp}(a))$, *then* $\mathtt{s}(a) > \mathtt{s}(b)$. ***(Strict Monotony)***

Consideration (**Dis**) states that dissimilarities should be taken into account. This amount to ensuring that attackers have impact on their targets. The following weakening principle captures this idea. It extends the one from [4] and states that if attackers overcome supporters, the argument should lose weight.

Principle 4 *(Weakening) A semantics* $\mathbf{S}$ *satisfies* weakening *iff for any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$*, for any* $a \in \mathcal{A}$ *such that there exists an injective function* f *from* $\mathtt{Supp}(a)$ *to* $\mathtt{Att}(a)$ *such that*

- $\forall x \in \mathtt{Supp}(a)$, $\mathtt{s}(x) \leq \mathtt{s}(f(x))$ *and* $\delta((x,a)) \leq \delta((f(x),a))$,
- $\forall X \subseteq \mathtt{Supp}(a)$, $\sigma(X) \leq \sigma(\{f(x) \mid x \in X\})$,

the following holds:

- $\mathtt{s}(a) \leq \sigma(\{a\})$ ***(Weakening)***
- *if* $(\sigma(\{a\}) > 0)$ *and* $(\{b \in \mathtt{Att}(a) \mid \delta((b,a)) \times \mathtt{s}(b) \neq 0\} \setminus \{f(x) \mid x \in \mathtt{Supp}(a)\} \neq \emptyset$ *or* $\exists x \in \mathtt{Supp}(a)$ *s.t* $\mathtt{s}(x) < \mathtt{s}(f(x))$ *or* $\exists X \subseteq \mathtt{Supp}(a)$ *s.t* $\sigma(X) < \sigma(\{f(x) \mid x \in X\}))$, *then* $\mathtt{s}(a) < \sigma(\{a\})$ ***(Strict Weakening)***

Several weighting semantics have been proposed in the literature. A few of them deal with support graphs, some with attack graphs, and finally a few with bipolar graphs. None of this plethora of semantics deals with interactions between arguments as they all assume that only individual arguments may have basic weights. However, interactions may exist in case of analogical arguments, and are important for argument strength as suggested by the diversity (**Div**) consideration. Let us illustrate how two existing semantics may lead to inaccurate assessments of analogical arguments.

The first one is weighted h-Categorizer that have been studied in [**?**]. This semantics takes as input an AF $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$, where the support relation is empty ($\mathcal{R}_s = \emptyset$), σ is defined from $\mathcal{A}$ to $[0,1]$, and assigns to every argument $a \in \mathcal{A}$ a value as follows:

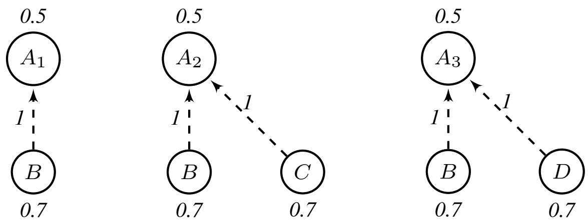
$$\mathtt{s}^h(a) = \frac{\sigma(a)}{1 + \sum\limits_{b \mathcal{R}_a a} \delta((b,a)) \times \mathtt{s}^h(b)} \qquad (1)$$

Example 2 *In our running example, assume that* A *has no supporters and it is only attacked by* E *and* F*. Consider the functions* π, δ *as described previously and recall that* $\sigma(\{A\}) = 0.5$ *and* $\sigma(\{E\}) = \sigma(\{F\}) = \sigma(\{E,F\}) = 0.8$*. Note that having both features* e *and* s *has the same impact as having only one of them. This means that the two features are redundant, and thus only one of them should be considered. The above semantics will unfortunately punish the argument* A *more than necessary since it will consider each attacker separately and neglect the interaction between the two. It will return the following scores:* $\mathtt{s}^h(E) = \mathtt{s}^h(F) = 0.8$ *and* $\mathtt{s}^h(A) = \frac{0.5}{1+0.7\times0.8+0.2\times0.8} = 0.29$.

Let us consider the *Aggregation-based* (Agg) semantic from [3]. It was defined for support graphs, i.e., AF $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$ where the attack relation is empty ($\mathcal{R}_a = \emptyset$), σ is defined from $\mathcal{A}$ to $[0,1]$ and $\delta \equiv 1$, i.e., all relations are assumed to be fully relevant. Agg assigns to every $a \in \mathcal{A}$ a value as follows:

$$\mathtt{s}^g(a) = \sigma(a) + (1 - \sigma(a)) \times \frac{\sum\limits_{b \mathcal{R}_s a} \mathtt{s}^g(b)}{1 + \sum\limits_{b \mathcal{R}_s a} \mathtt{s}^g(b)} \qquad (2)$$

Example 3 *Consider the three graphs depicted below and assume that* B *and* C *promote two features on the same topic while* D *refers to a feature of another kind. Assume also that* $\sigma(\{B,C\}) = 0.7$ *and* $\sigma(\{B,D\}) = 0.9$*. As in the previous example,* B *is redundant with* C*. Thus, intuitively one would expect that* A_1 *should have the same strength as* A_2*. Despite the fact that all supporters have the same basic weight in the 3 graphs, the group* $\{B,D\}$ *supporting* A_3 *is stronger than the groups supporting the other arguments. Thus, one expects* A_3 *to be stronger than* A_1, A_2*. However, the aggregation-based semantics returns the following scores:* $\mathtt{s}^g(B) = \mathtt{s}^g(C) = \mathtt{s}^g(D) = 0.7$ *in the three graphs while* $\mathtt{s}^g(A_1) = 0.70$, $\mathtt{s}^g(A_2) = 0.79$*, and* $\mathtt{s}^g(A_3) = 0.79$*. Note that this semantics declares* A_2 *as stronger than* A_1 *because it does not take into account the redundancy that exists between the arguments* B, C*. It does not either consider the synergy that exists between* B, D *and declares* A_2 *as strong as* A_3*, while it should not be the case.*



To sum up, existing weighting semantics are unable to capture possible interactions (synergies, redundancies) that may exist between attackers/supporters. Some of them like Agg do not either deal with varied-strength attacks/supports.

5 CHOQUET-BASED SEMANTICS

In what follows, we present novel semantics that satisfy the principles discussed in Section 4 and that deal properly with the capacity over the set of features. We start first by extending both weighted h-Categorizer semantics and aggregation-based semantics, then we propose another semantics for bipolar argumentation frameworks. The basic idea behind the three semantics is to use Choquet integral [11] for aggregating the strengths of attackers (respectively supporters)of an argument. Choquet integral is an aggregation function that is defined as follows:

The *Choquet integral* of $x = (x_1, \ldots, x_n) \in \Re^n$ wrt a capacity μ on a set $N = \{1, \ldots, n\}$ is:

$$C(x) = \sum_{i=1}^{n} x_{\nu(i)}[\mu(A_{\nu(i)}) - \mu(A_{\nu(i+1)})],$$

where ν is a permutation on N such that $x_{\nu(1)} \leq \ldots, \leq x_{\nu(n)}$, $A_{\nu(i)} = \{\nu(i), \ldots, \nu(n)\}$ for every $i \in N$, and $A_{\nu(n+1)} = \emptyset$.

The difference $\mu(A_{\nu(i)}) - \mu(A_{\nu(i+1)})$ represents the *marginal importance* of element $i \in N$.

Example 4 *Let us illustrate this function with an example where* $N = \{1, 2, 3\}$ *and* $\mu(\{1\}) = \mu(\{2\}) = \mu(\{3\}) = \mu(\{1, 2\}) = 0.7$, $\mu(\{1, 3\}) = \mu(\{2, 3\}) = 0.9$, *and* $\mu(\{1, 2, 3\}) = 1$. *Let* $x = (0.7, 0.7, 0.7)$. *Here the three values are equal, thus there is no need for rank-ordering them from the weakest value to the strongest one. It is easy to check that* $C(x) = 0.7(\mu(\{1, 2, 3\} - \mu(\{2, 3\}) + 0.7(\mu(\{2, 3\} - \mu(\{3\}) + 0.7(\mu(\{3\} - \mu(\{\}) = 0.7 \times 0.1 + 0.7 \times 0.2 + 0.7 \times 0.7 = 0.7$.

5.1 Extending weighted h-categorizer

Throughout this section, we assume that the support relation is empty ($\mathcal{R}_s = \emptyset$). For capturing interactions between attackers, we replace in Equation 1 the function that aggregates the attackers, which is the sum of all attackers scores, by Choquet Integral.

Definition 5 (Choquet-based Categorizer) Choquet-based Categorizer *is a function transforming any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s = \emptyset, \mathcal{R}_a, \delta \rangle$ *into a function* $\mathtt{s}_c^h$ *from* $\mathcal{A}$ *to* $[0, 1]$ *s.t for any* $a \in \mathcal{A}$,

$$\mathtt{s}_c^h(a) = \frac{\sigma(\{a\})}{1 + \sum_{i=1}^{n} x_{\nu(i)} \times [\sigma(A_{\nu(i)}) - \sigma(A_{\nu(i+1)})]}$$

where ν *is a permutation on* $\mathtt{Att}(a) = \{b_1, \ldots, b_n\}$ *such that* $x_{\nu(1)} \leq \ldots \leq x_{\nu(n)}$ *and* $x_{\nu(i)} = \delta((b_{\nu(i)}, a)) \times \mathtt{s}_c^h(b_{\nu(i)})$, $A_{\nu(i)} = \{b_{\nu(i)}, \ldots, b_{\nu(n)}\}$ *and* $A_{\nu(n+1)} = \emptyset$.
If $\mathtt{Att}(a) = \emptyset$, *then* $\sum_{i=1}^{n} x_{\nu(i)} \times [\sigma(A_{\nu(i)}) - \sigma(A_{\nu(i+1)})] = 0$.

It is easy to check that if an argument is not attacked, then its strength is equal to its basic weight. Furthermore, when an argument is attacked by at least one argument whose strength is not 0, then the argument looses weight (i.e., its strength is less than its basic weight).

Example 2 (Cont) Consider again the argument A_6 from Figure 1 with $\sigma(\{A\}) = 0.5$ and $\sigma(\{E\}) = \sigma(\{F\}) = \sigma(\{E, F\}) = 0.8$. Since E and F are not attacked, then $\mathtt{s}_c^h(E) = \mathtt{s}_c^h(F) = 0.8$. Regarding A_6, note that $\mathtt{s}_c^h(E) \times \delta((E, A_6)) = 0.8 \times 0.7 = 0.56$ and $\mathtt{s}_c^h(F) \times \delta((F, A_6)) = 0.8 \times 0.2 = 0.16$. Hence, we consider the ordering $\langle F, E \rangle$ and so $\mathtt{s}_c^h(A_6) = \frac{0.5}{1+0.16\times[\sigma(\{F,E\})-\sigma(\{E\})]+0.56\times[\sigma(\{E\})-\sigma(\emptyset)]} = 0.345$. It is worth noticing that the attacker F is not taken into account since it is redundant with E. This is captured by its marginal importance given by $\sigma(\{F, F\}) - \sigma(\{E\}) = 0$. Consequently, the argument A_6 is less punished and its score is greater than the one it received by Weighted h-Categorizer (recall that it was 0.29).

It is worth mentioning that assigning scores to arguments with the above semantics amounts mainly to solving a system of equations (one per argument) for each argumentation graph. In what follows, we show that each such system has a unique solution whatever the topology of the graph. Indeed, even in presence of cycles, this semantics guarantees a unique score to every argument.

Theorem 1 *For any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s = \emptyset, \mathcal{R}_a, \delta \rangle$, *Choquet-based Categorizer assigns a unique value to each argument* $a \in \mathcal{A}$.

This shows that $\mathtt{s}_c^h$ is well-defined. The next result shows that it satisfies the four principles introduced in the previous section.

Theorem 2 *Choquet-based Categorizer satisfies the large and strict versions of all the principles.*

5.2 Extending aggregation-based semantics

Throughout this section, we assume that the attack relation is empty. We empower the semantics $\mathtt{Agg}$, recalled in Equation 2, to deal with varied-strengths support relations and with possible interactions between supporters. The novel semantics is defined as follows:

Definition 6 (Choquet-based $\mathtt{Agg}$) Choquet-based $\mathtt{Agg}$ *is a function transforming any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a = \emptyset, \delta \rangle$ *into a function* $\mathtt{s}_c^g$ *from* $\mathcal{A}$ *to* $[0, 1]$ *s.t for any* $a \in \mathcal{A}$,

$$\mathtt{s}_c^g(a) = \sigma(\{a\} + (1 - \sigma(\{a\}) \times X$$

where

$$X = \frac{\sum_{i=1}^{n} \delta((b_{\nu(i)}, a)) \times \mathtt{s}_c^g(b_{\nu(i)}) \times [\sigma(A_{\nu(i)}) - \sigma(A_{\nu(i+1)})]}{1 + \sum_{i=1}^{n} \delta((b_{\nu(i)}, a)) \times \mathtt{s}_c^g(b_{\nu(i)}) \times [\sigma(A_{\nu(i)}) - \sigma(A_{\nu(i+1)})]}$$

where ν *is a permutation on* $\mathtt{Supp}(a) = \{b_1, \ldots, b_n\}$ *such that* $\delta((b_{\nu(1)}, a)) \times \mathtt{s}_c^g(b_{\nu(1)}) \leq \ldots \leq \delta((b_{\nu(n)}, a)) \times \mathtt{s}_c^g((b_{\nu(n)})$, $A_{\nu(i)} = \{b_{\nu(i)}, \ldots, b_{\nu(n)}\}$ *and* $A_{\nu(n+1)} = \emptyset$.

If an argument is not supported, then its strength is equal to its basic weight. When an argument is supported by at least one argument whose strength is not 0, then the strength of the argument increases (i.e., its strength is greater than its basic weight).

Let us illustrate the new semantics with an example.

Example 3 (Cont) Consider the three graphs of Example 3. Recall that $\sigma(\{B, C\}) = 0.7$ and $\sigma(\{B, D\}) = 0.9$. Since the arguments B, C, D are not supported, then $\mathtt{s}_c^g(B) = 0.7$ in in the three graphs, $\mathtt{s}_c^g(C) = \mathtt{s}_c^g(D) = 0.7$. $\mathtt{s}_c^g(A_1) = 0.5 + 0.5 \times \frac{0.7\times[\sigma(\{B\})-\sigma(\{\})]}{1+0.7\times[\sigma(\{B\})-\sigma(\{\})]} = 0.66$ $\mathtt{s}_c^g(A_2) = 0.5 + 0.5 \times \frac{0.7\times[\sigma(\{B,C\})-\sigma(\{C\})]+0.7\times[\sigma(\{C\})-\sigma(\{\})]}{1+0.7\times[\sigma(\{B,C\})-\sigma(\{C\})]+0.7\times[\sigma(\{C\})-\sigma(\{\})]} = 0.66$. Finally, $\mathtt{s}_c^g(A_3) = 0.5 + 0.5 \times \frac{0.7\times[\sigma(\{B,D\})-\sigma(\{D\})]+0.7\times[\sigma(\{D\})-\sigma(\{\})]}{1+0.7\times[\sigma(\{D,C\})-\sigma(\{D\})]+0.7\times[\sigma(\{D\})-\sigma(\{\})]} = 0.69$. It is worth noticing that the new semantics handles properly the interactions between B, C. Indeed, since $\sigma(\{B, C\}) = \sigma(\{B\})$, then adding C to B does not add anything. Thus, the new semantics considers that the group supporting A_1 is as strong as the one supporting A_2. Recall that the original version of this semantics assigns a greater score to A_2, neglecting thus the negative interaction (redundancy) between B, C.

As for Choquet-based Categorizer, we show that the extended version of Aggregation-based semantics assigns a unique score to every argument. In other words, the system of equations associated to arguments has always a solution, and it is unique.

Theorem 3 *For any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a = \emptyset, \delta \rangle$, *Choquet-based* $\mathtt{Agg}$ *assigns a unique value to each argument* $a \in \mathcal{A}$.

This shows that $\mathtt{s}_c^g$ is well-defined. The next result shows that it satisfies the four principles introduced in the previous section.

Theorem 4 *Choquet-based* $\mathtt{Agg}$ *semantics satisfies the large and strict versions of all the principles.*

5.3 Extending euler-based semantics

In the computational argumentation literature, several weighting semantics have been proposed for unipolar argumentation frameworks, and mostly for graphs where only attacks are available. The reasons are twofold: First, attack graphs are suitable for conflict resolution problems. Second, as shown in [17], finding semantics that ensure unique value for every argument in a bipolar graph is not an easy task, especially when the graph contains cycles. For instance, the authors have shown that a semantics that uses the *sum* operator for aggregating separately the attackers and the supporters of an argument does not guarantee uniqueness of solutions. Consequently, in [4, 24], the authors proposed semantics for acyclic graphs only. In what follows, we extend Euler-based semantics from [4] as it satisfies more properties. This semantics is defined as follows: For every acyclic graph $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$, with $\delta \equiv 1$ and σ be a function from $\mathcal{A}$ to $[0, 1]$, for every $a \in \mathcal{A}$,

$$\mathtt{s}^e(a) = 1 - \frac{1-\sigma(a)^2}{1+\sigma(a)\mathtt{e}^E}, \quad \text{where}$$

$$E = \sum_{x \in \mathtt{Supp}(a)} \mathtt{s}^e(x) - \sum_{y \in \mathtt{Att}(a)} \mathtt{s}^e(y).$$

Extending Euler-based semantics amounts to replacing both sum functions that aggregate supports/attacks by Choquet integral. Furthermore, the novel semantics will deal with weighted relations for capturing the relevance consideration (**Rel**) discussed previously.

Definition 7 (Extended Euler-based Semantics) *Extended Euler-based is a function transforming any AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$ *into a function* $\mathtt{s}^e_c$ *from* $\mathcal{A}$ *to* $[0, 1]$ *s.t for any* $a \in \mathcal{A}$,

$$\mathtt{s}^e_c(a) = 1 - \frac{1-\sigma(\{a\})^2}{1+\sigma(\{a\})\mathtt{e}^E}, \quad \textit{where}$$

$$E = E_1 - E_2$$

$$E_1 = \sum_{i=1}^{n} \delta((s_{\nu(i)}, a)) \times \mathtt{s}^e_c(s_{\nu(i)}) \times [\sigma(A_{\nu(i)}) - \sigma(A_{\nu(i+1)})],$$

$$E_2 = \sum_{i=1}^{m} \delta((a_{\phi(i)}, a)) \times \mathtt{s}^e_c(a_{\phi(i)}) \times [\sigma(A_{\phi(i)}) - \sigma(A_{\phi(i+1)})].$$

where ν *is a permutation on* $\mathtt{Supp}(a) = \{s_1, \ldots, s_n\}$ *such that* $\delta((s_{\nu(1)}, a)) \times \mathtt{s}^e_c(s_{\nu(1)}) \leq \ldots \leq \delta((s_{\nu(n)}, a)) \times \mathtt{s}^e_c((s_{\nu(n)}),$ ϕ *is a permutation on* $\mathtt{Att}(a) = \{a_1, \ldots, a_m\}$ *such that* $\delta((a_{\phi(1)}, a)) \times \mathtt{s}^e_c(a_{\phi(1)}) \leq \ldots \leq \delta((a_{\phi(m)}, a)) \times \mathtt{s}^e_c((a_{\phi(m)}),$ $A_{\nu(i)} = \{s_{\nu(i)}, \ldots, s_{\nu(n)}\}$, $A_{\nu(n+1)} = \emptyset$, $A_{\phi(i)} = \{a_{\phi(i)}, \ldots, a_{\phi(m)}\}$ *and* $A_{\phi(m+1)} = \emptyset$.

It is easy to check that if an argument is not attacked and not supported, then its strength is equal to its basic weight.

Let us illustrate how this semantics will evaluate the arguments of our running example.

Example 1 (Cont) Recall that: $\sigma(\{B, C\}) = 0.75$, $\sigma(\{B, D\}) = 0.9$, $\sigma(\{C, D\}) = 0.9$, $\sigma(\{B, C, D\}) = 0.9$, $\sigma(\{E, F\}) = 0.8$. It is easy to check that $\mathtt{s}^e_c(B) = \mathtt{s}^e_c(C) = \mathtt{s}^e_c(D) = 0.7$, $\mathtt{s}^e_c(E) = \mathtt{s}^e_c(F) = 0.8$, and $E_1 = 0.357$, $E_2 = 0.448$, and $\mathtt{s}^e_c(A) = 0.48$. Note that the argument A is weakened (its basic weight was 0.5 and its final strength is 0.48). This is due to its group of supporters which is weaker than the group of attackers.

This semantics guarantees a unique assignment of values to every argument of a given framework when the latter does not contain cycles. Unfortunately, this is not the case when cycles are present.

Theorem 5 *For any acyclic AF* $\mathbf{A} = \langle \mathcal{A}, v, \sigma, \mathcal{R}_s, \mathcal{R}_a, \delta \rangle$*, the extended Euler-based semantics assigns a unique value to each argument* $a \in \mathcal{A}$*.*

The semantics satisfies the four principles introduced in Section 3.

Theorem 6 *Extended Euler-based semantics satisfies the large and strict versions of all the principles.*

6 RELATED WORK

As said before, several philosophers have tackled the question of evaluation of analogical arguments. However, none of them have proposed formal semantics. In the computational argumentation literature, to the best of our knowledge, there are two works on the question. The first one [8] has used extension semantics from [12]. However, our running example shows that such semantics are not suitable for analogical arguments. The second work [23] has used the logical language of the ABA framework for representing analogical arguments, but did not focus on their evaluation. Besides, there is a plethora of semantics in the literature, and most of them have been compared in [5] wrt a set of principles. Some of the semantics that satisfy desirable properties, namely Strict Monotony and Strict Weakening, use the *max* operator for aggregating attackers/supporters. This means that among all attackers (resp. supporters), the operator considers only the strongest one. Such semantics are definitely not suitable in case of analogical arguments, where the numbers of highlighted similarities and dissimilarities are crucial.

The Strict Monotony principle suggests *accrual* of attackers/supporters of any analogical argument. In [16, 22], a particular form of accrual has been studied. It states that the more a claim is supported by reasons, the more likely it is. Hence, the focus is on arguments having the same conclusion. It is worth mentioning that the supporters (resp. attackers) of the same argument do not necessarily have the same conclusion. In [22], three principles were presented and deemed as mandatory for any formal treatment of accrual. They state the following: (P1) “an accrual may be weaker than its elements”, (P2) “an accrual makes its elements inapplicable”, and (P3) “flawed arguments are not accrued”. While these principles are satisfied by the approach studied in [16], (P1) and (P2) are not applicable for analogical arguments. Indeed, the strength of a group, like $\{B, C, D\}$ in the running example, is at least as strong as any of its elements. This is in accordance with the requirement (Num) encoded by the Monotony principle, which is itself rooted in the monotonicity of the capacity defined on the set of features. (P3) is compatible with the idea that attackers/supporters of strength 0 have no impact. This property is satisfied by our three semantics.

7 CONCLUSION

Starting from works by philosophers on the evaluation of analogical arguments, we proposed a bipolar framework for reasoning about such arguments. Its relations and (groups of) arguments are weighted. A characteristic of analogical arguments is that their attackers (resp. supporters) may interact. However, these interactions are not considered by any existing semantics. The paper filled the gap by proposing three semantics. They use Choquet integral for aggregating attacks/supports, and are shown to satisfy desirable principles.

This work can be extended in several ways. The most urgent one is to solve the problem of bipolar graphs with cycles. Another future work consists of comparing our approach with works on case-based reasoning (CBR). Our semantics evaluate somehow to what extent two cases (a past case and a new one) are similar. A starting point for a formal comparison would be comparing the "argumentation-based similarity measures" with measures used by CBR models.

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